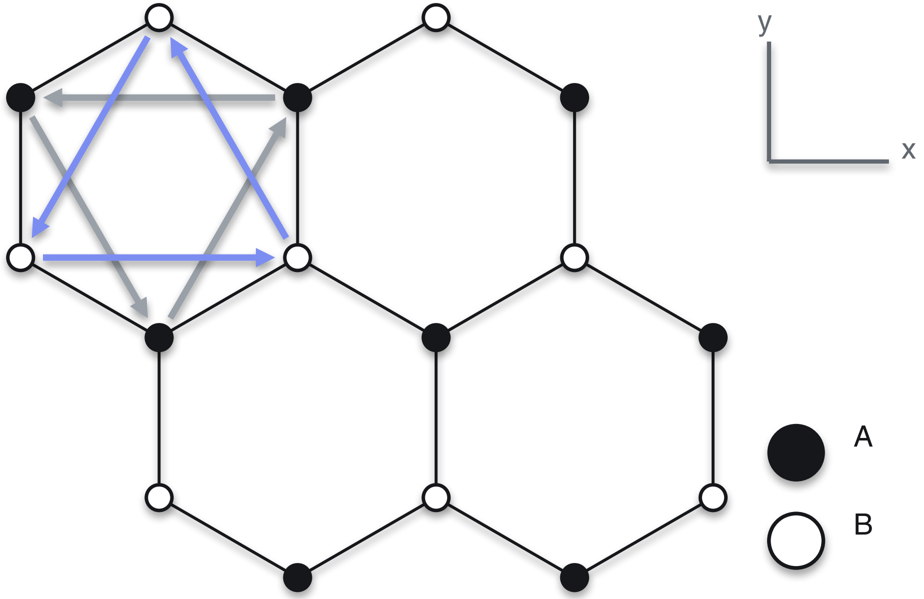
All the material from these lectures will be here:

Quantum Hall physics without Landau levels

The Haldane and Kane–Mele models on the honeycomb lattice



Luis E. F. Foà Torres · Universidad de Chile

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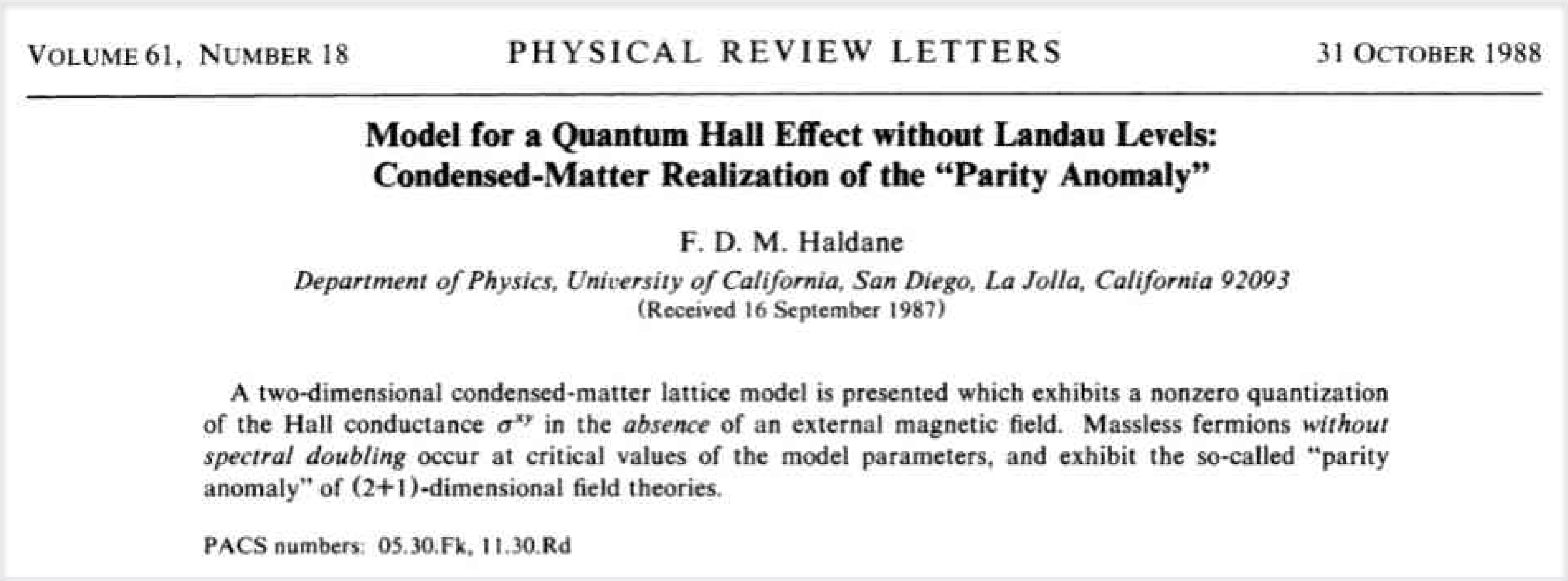
Model for a quantum Hall effect without Landau levels

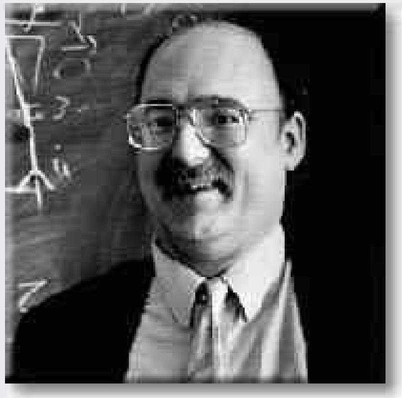

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane
Department of Physics, University of California, San Diego, La Jolla, California 92093
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.

PACS numbers: 05.30.Fk, 11.30.Rd



F. D. M. Haldane

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988)

Model for a quantum Hall effect without Landau levels

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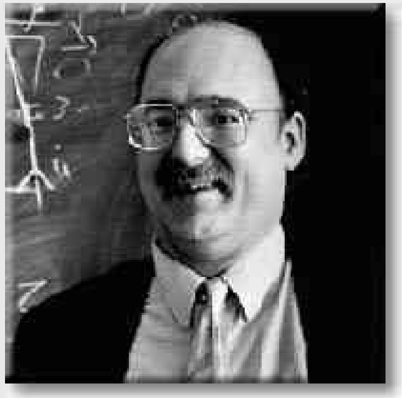


F. D. M. Haldane

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988)

A quantized Hall conductance — with no external magnetic field.

Symmetries of graphene

INVERSION / SUBLATTICE SYMMETRY

like the chiral symmetry of the SSH model, it satisfies

$$\sigma_z \, \mathcal{H}_{\mathbf{k}} \, \sigma_z = -\mathcal{H}_{\mathbf{k}}$$

The Bloch Hamiltonian is block off-diagonal with respect to the two sublattices.

Symmetries of graphene

TIME-REVERSAL SYMMETRY (TRS)

we do not consider spin here — time reversal is just complex conjugation

$$\mathcal{H}_{\mathbf{k}} = \mathcal{H}^*_{-\mathbf{k}}$$

$K_+ \;\rightarrow\; K_-$ time reversal exchanges the two valleys

Symmetries of graphene

THREE-FOLD ROTATION SYMMETRY

Remember its consequences — e.g. trigonal warping of the bands.

Symmetries of graphene

IS

$$\sigma_z \mathcal{H}_{\mathbf{k}} \sigma_z = -\mathcal{H}_{\mathbf{k}}$$

TRS

$$\mathcal{H}_{\mathbf{k}} = \mathcal{H}^*_{-\mathbf{k}}$$

C3

three-fold rotation symmetry

Symmetries of graphene

IS

$$\sigma_z \mathcal{H}_{\mathbf{k}} \sigma_z = -\mathcal{H}_{\mathbf{k}}$$

TRS

$$\mathcal{H}_{\mathbf{k}} = \mathcal{H}^*_{-\mathbf{k}}$$

C3

three-fold rotation symmetry

To open a gap we must break either IS or TRS.

Symmetries of graphene

IS $$\sigma_z \mathcal{H}_{\mathbf{k}} \sigma_z = -\mathcal{H}_{\mathbf{k}}$$

TRS $$\mathcal{H}_{\mathbf{k}} = \mathcal{H}^*_{-\mathbf{k}}$$

C3 three-fold rotation symmetry

A gap is necessary: a single band carries no Berry curvature (remember the sum rule).

To open a gap we must break either IS or TRS.

Adding a mass term — breaking sublattice symmetry

Let the A and B sites have different on-site energies, say $+\Delta$ and $-\Delta$.

$$\varepsilon_{\mathbf{k}} = \pm\sqrt{|h(\mathbf{k})|^2 + \Delta^2}$$

The spectrum acquires a gap — but no edge states.

☹

Adding a mass term — breaking sublattice symmetry

Let the A and B sites have different on-site energies, say $+\Delta$ and $-\Delta$.

$$\varepsilon_{\mathbf{k}} = \pm\sqrt{|h(\mathbf{k})|^2 + \Delta^2}$$

The spectrum acquires a gap — but no edge states. **Why?**

Adding a mass term — breaking sublattice symmetry

Let the A and B sites have different on-site energies, say $+\Delta$ and $-\Delta$.

$$\varepsilon_{\mathbf{k}} = \pm\sqrt{|h(\mathbf{k})|^2 + \Delta^2}$$

The spectrum acquires a gap — but no edge states. **Why?**

But wait — we also have the two golden rules for the Berry curvature!

Two golden rules for the Berry curvature

IS

$$\Omega(\mathbf{k}) = \Omega(-\mathbf{k})$$

TRS

$$\Omega(\mathbf{k}) = -\Omega(-\mathbf{k})$$

Two golden rules for the Berry curvature

IS

$$\Omega(\mathbf{k}) = \Omega(-\mathbf{k})$$

TRS

$$\Omega(\mathbf{k}) = -\Omega(-\mathbf{k})$$

Consequence: a non-vanishing Chern number requires broken time-reversal symmetry.

Two golden rules for the Berry curvature

IS

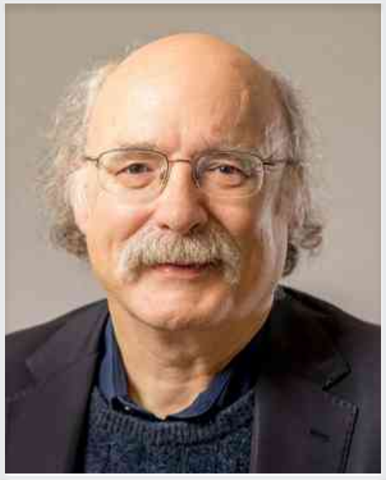
$$\Omega(\mathbf{k}) = \Omega(-\mathbf{k})$$

TRS

$$\Omega(\mathbf{k}) = -\Omega(-\mathbf{k})$$

Consequence: a non-vanishing Chern number requires broken TRS.

This can be done **with or without** a net magnetic flux.

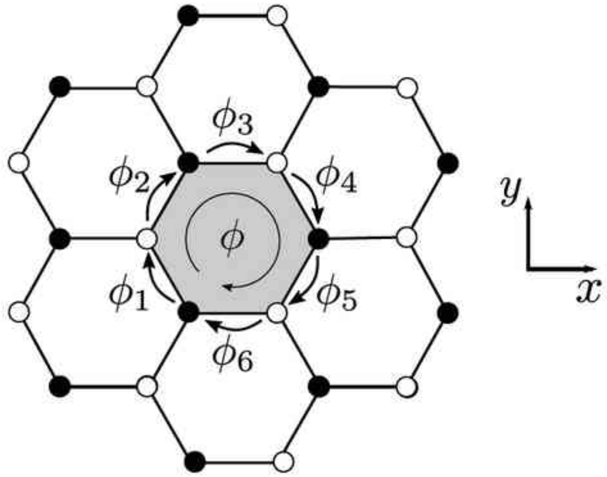


Adding a new mass term — breaking TRS

Right — we could add a Peierls phase…

But it must not come from a net magnetic field: that would just be the QH effect.

$$\gamma_{ij}(\boldsymbol{B}) = \exp\left(\mathrm{i}\,\frac{q}{\hbar}\int_{\boldsymbol{r}_i}^{\boldsymbol{r}_j} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{\ell}\right) \times \gamma_{ij}(\boldsymbol{B}=\boldsymbol{0})$$

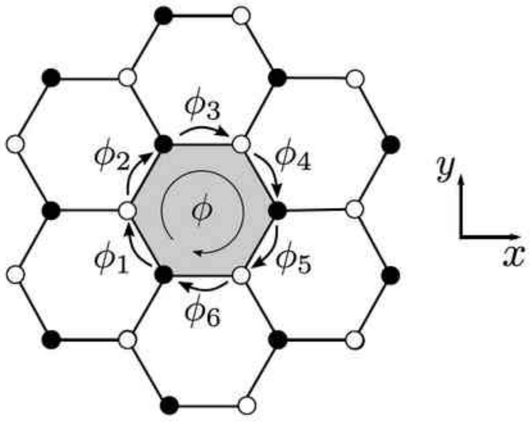


Adding a new mass term — breaking TRS

Right — we could add a Peierls phase…

But it must not come from a net magnetic field: that would just be the QH effect.

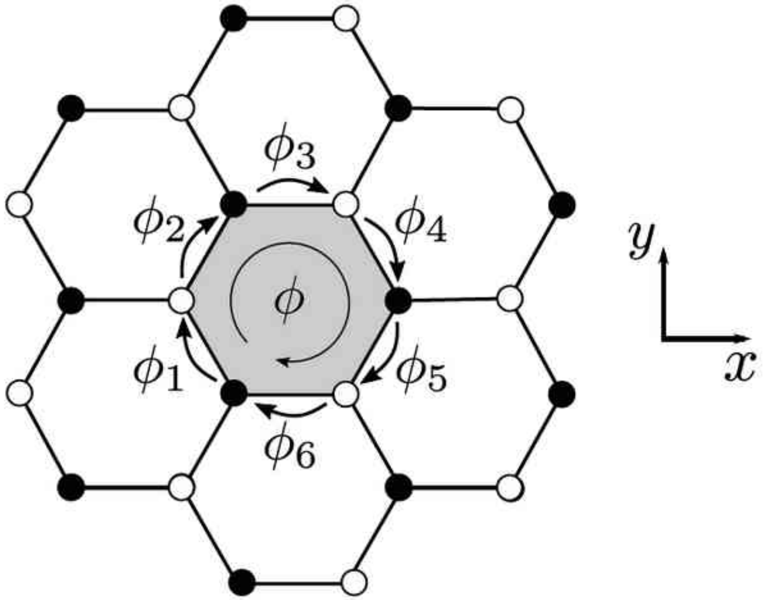
IDEA: keep the flux per plaquette equal to zero, while still having non-vanishing phases…
These phases act like local fields — enough to break TRS, but without producing a net flux.



Adding a new mass term — breaking TRS

IDEA: keep the flux per plaquette equal to zero, while still having non-vanishing phases… These phases act like local fields — enough to break TRS, but without producing a net flux.

But we cannot add such a phase to the nearest-neighbour hoppings!



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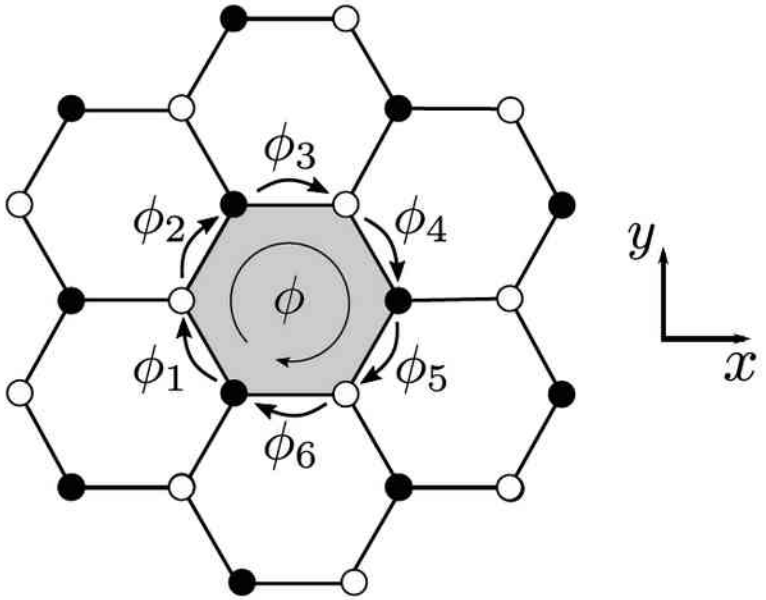
…so we go to **next**-nearest neighbours.



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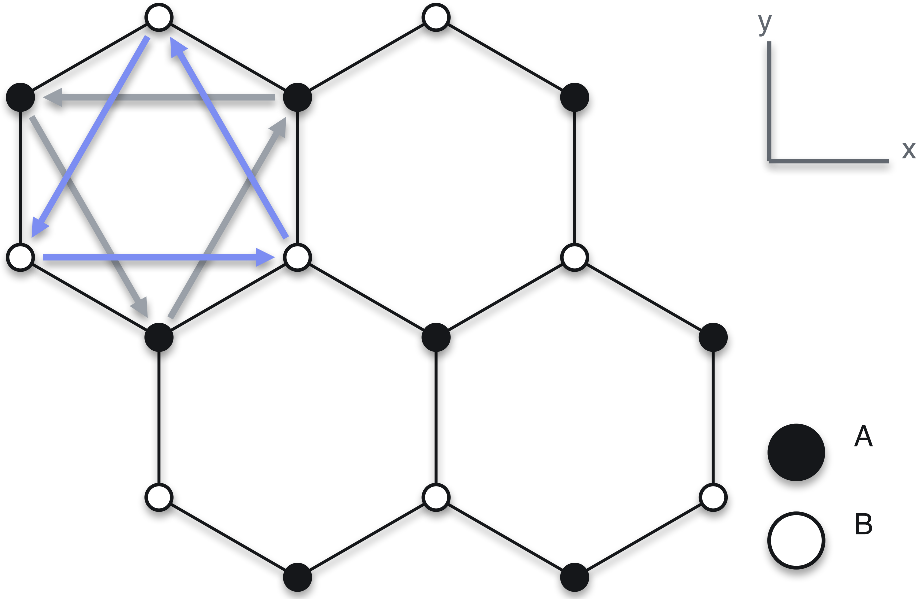


The Haldane model

$$\mathcal{H} = \Delta \sum_{j} \xi_j \, c_j^\dagger c_j + \gamma_0 \sum_{\langle i,j \rangle} c_i^\dagger c_j + \gamma_2 \sum_{\langle\langle i,j \rangle\rangle} e^{-\mathrm{i}\,\epsilon_{ij}\phi} \, c_i^\dagger c_j$$

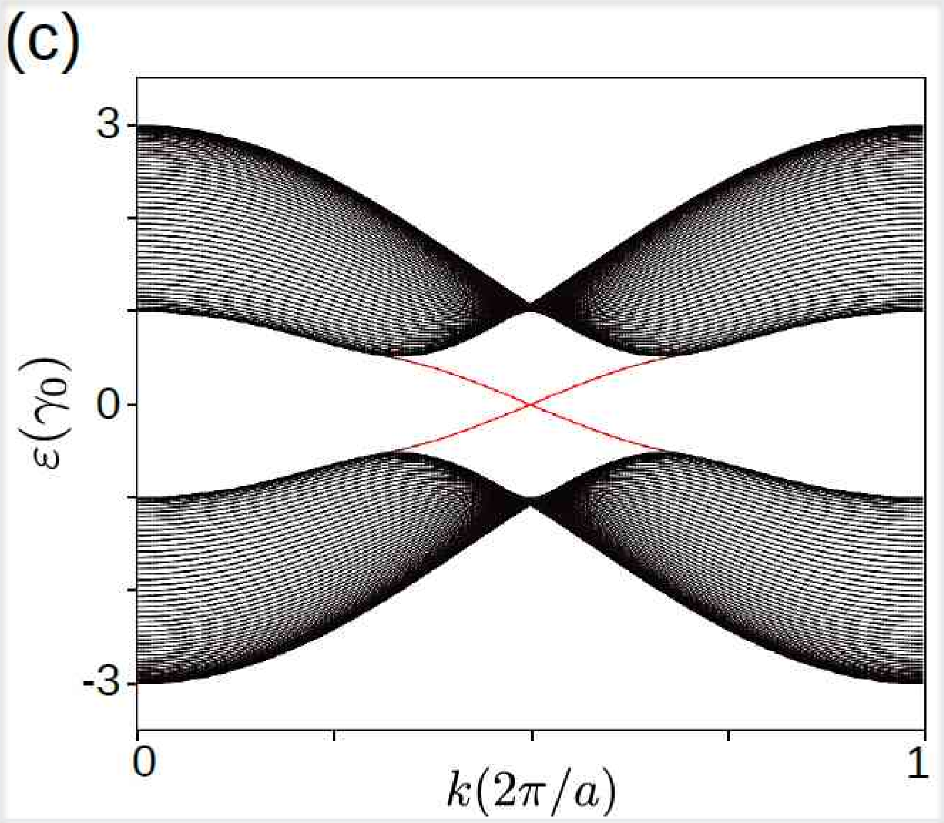
$\xi_j = +1$ for $j \in A$, $\xi_j = -1$ for $j \in B$

“mass” term breaking sublattice / inversion symmetry

Haldane term: second-nearest neighbours (complex!)

Gap + chiral edge states

$$\mathcal{H} = \Delta \sum_j \xi_j \, c_j^\dagger c_j + \gamma_0 \sum_{\langle i,j \rangle} c_i^\dagger c_j + \gamma_2 \sum_{\langle\langle i,j \rangle\rangle} e^{-\mathrm{i}\,\epsilon_{ij}\phi} \, c_i^\dagger c_j$$



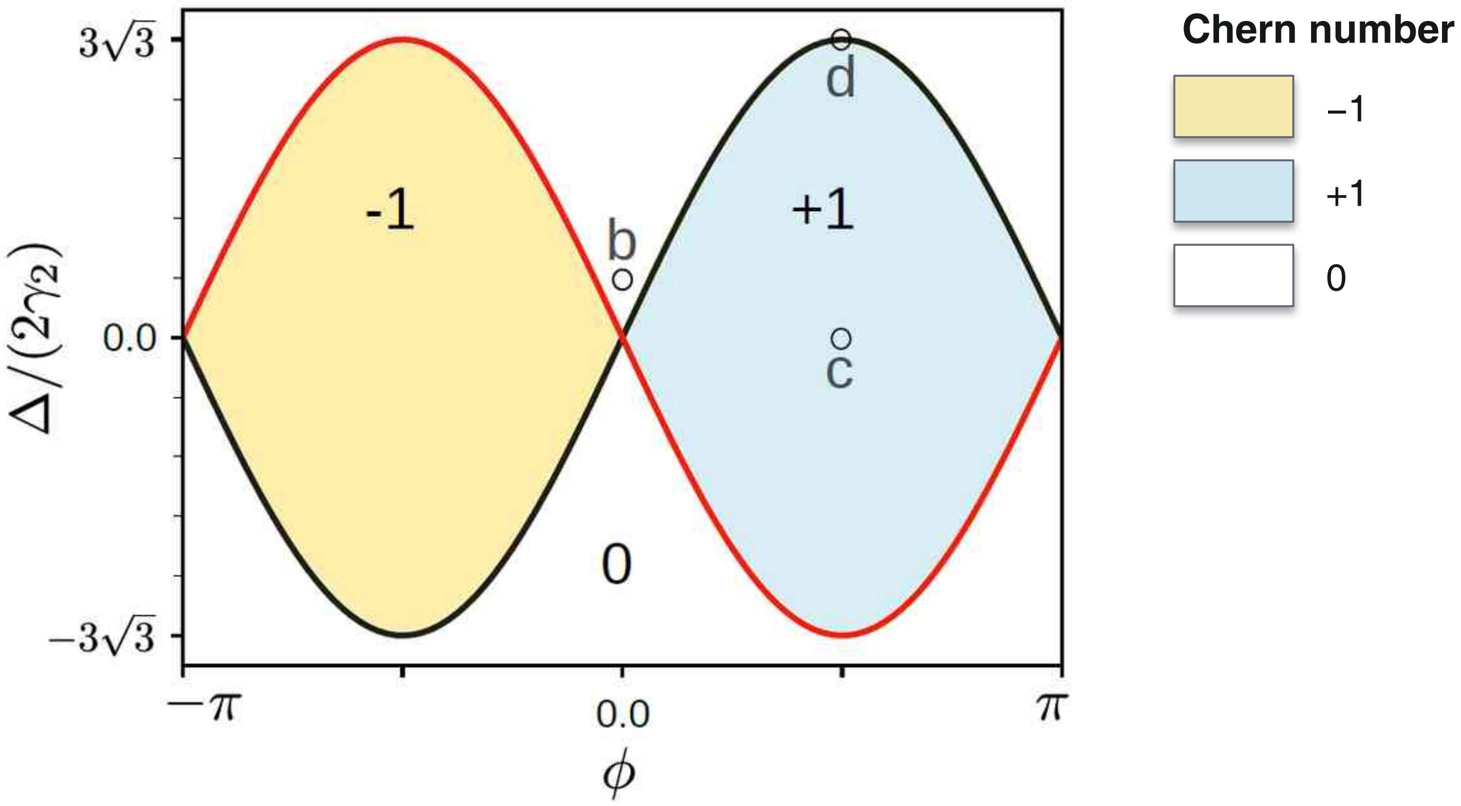
“Anomalous” Hall effect

The birth of the lattice Chern insulators

Gap + edge states
(one at each edge, ribbon geometry)

Foa Torres, S. Roche & J.-C. Charlier, Introduction to Graphene-Based Nanomaterials, 2nd ed., Cambridge University Press (2020)

The Haldane model: phase diagram



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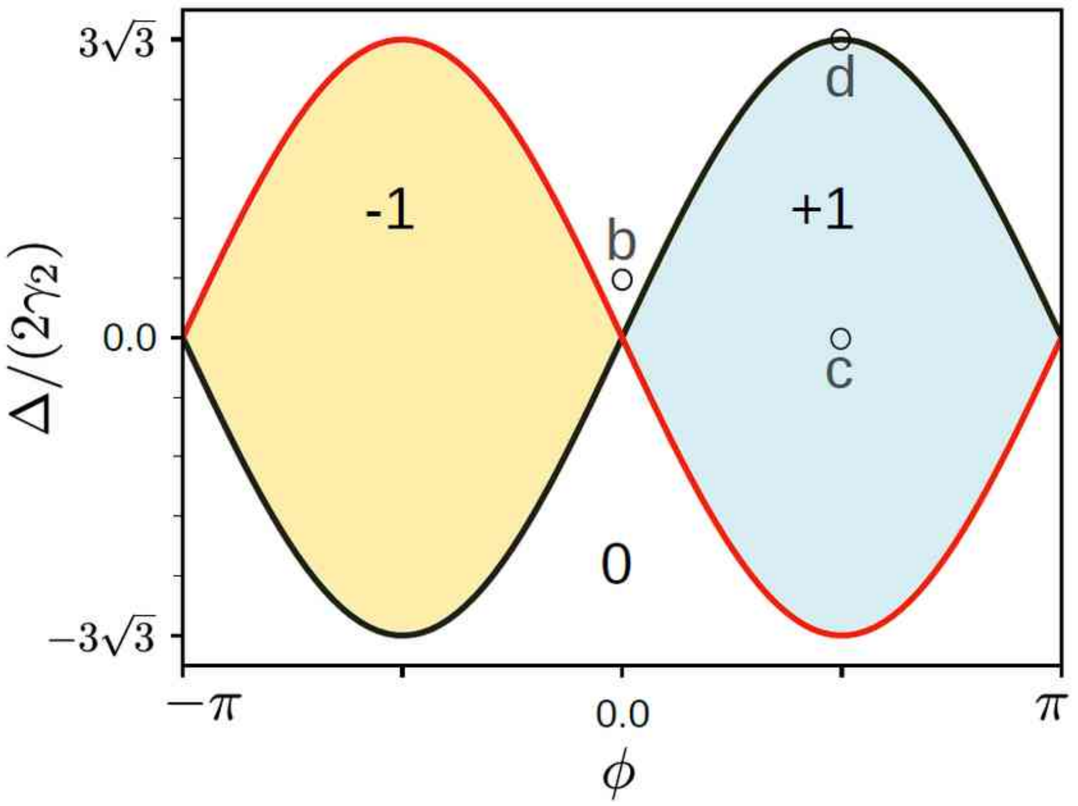

Chern number

- −1
- +1
- 0

The axes are the magnitudes controlling the two "mass terms".

Foa Torres, S. Roche & J.-C. Charlier, Introduction to Graphene-Based Nanomaterials, 2nd ed., Cambridge University Press (2020)

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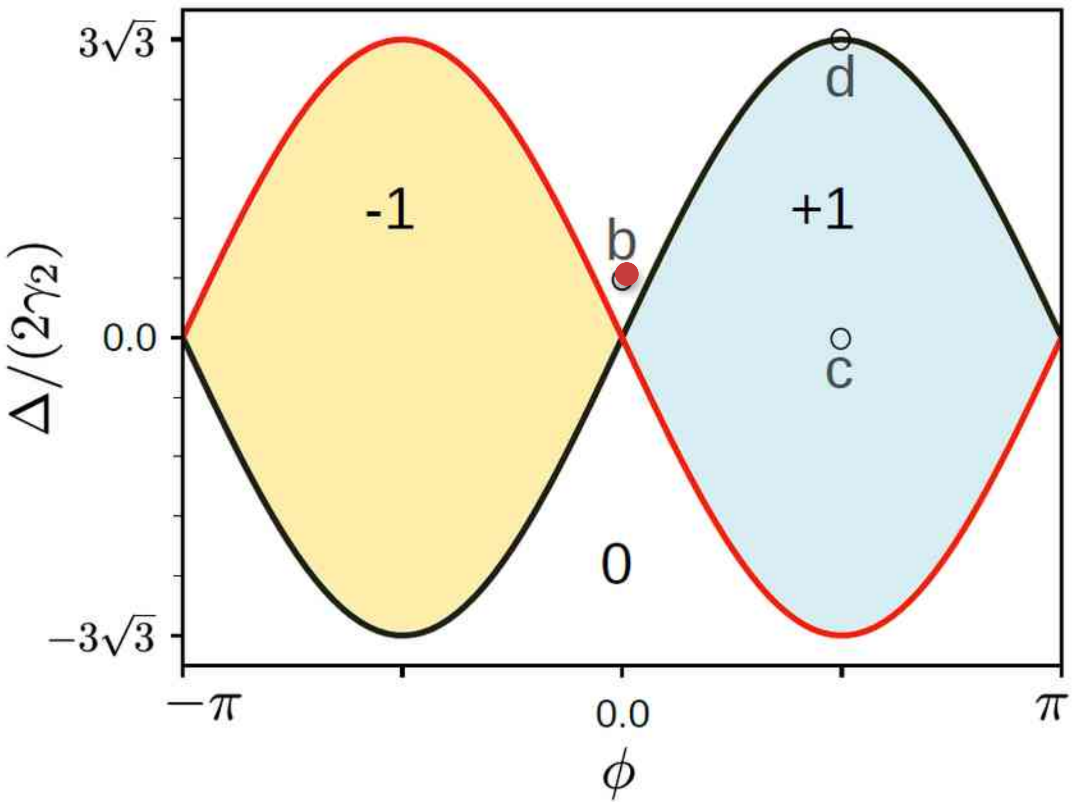

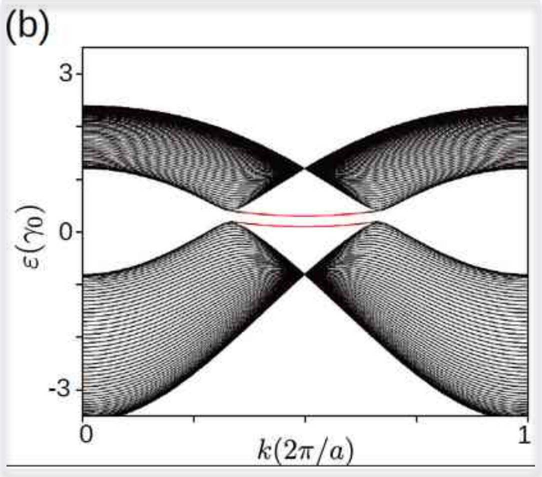

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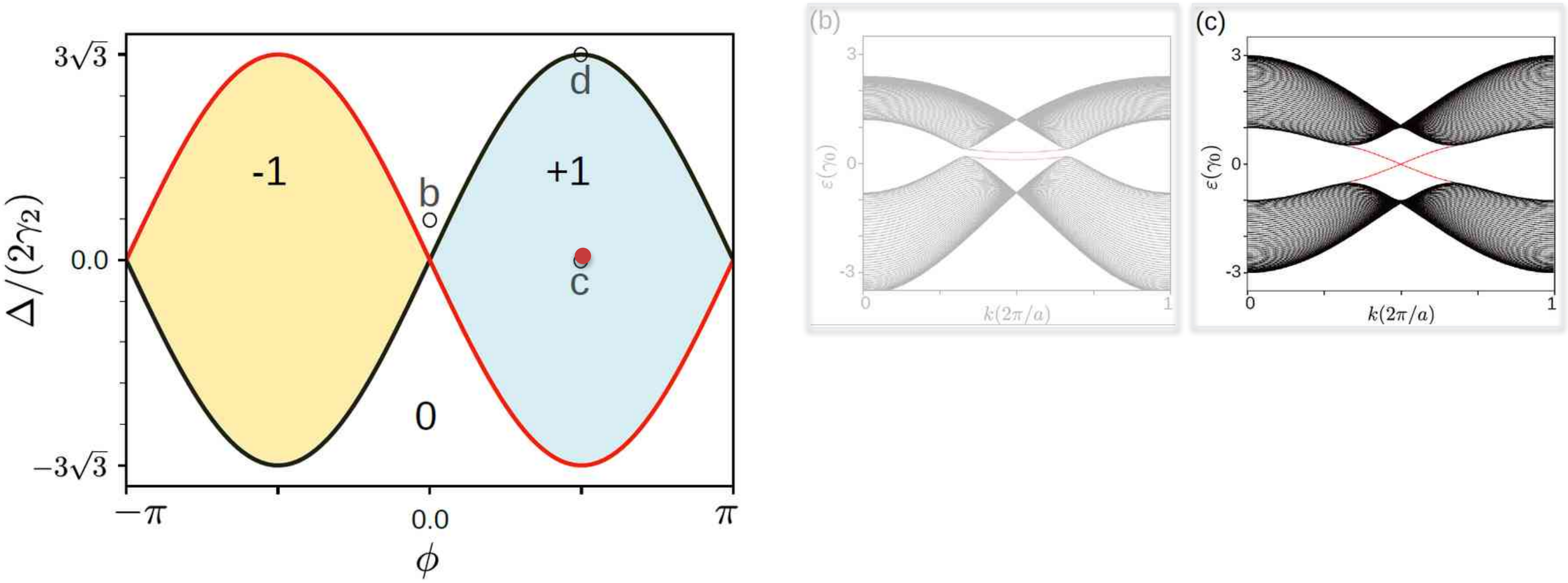

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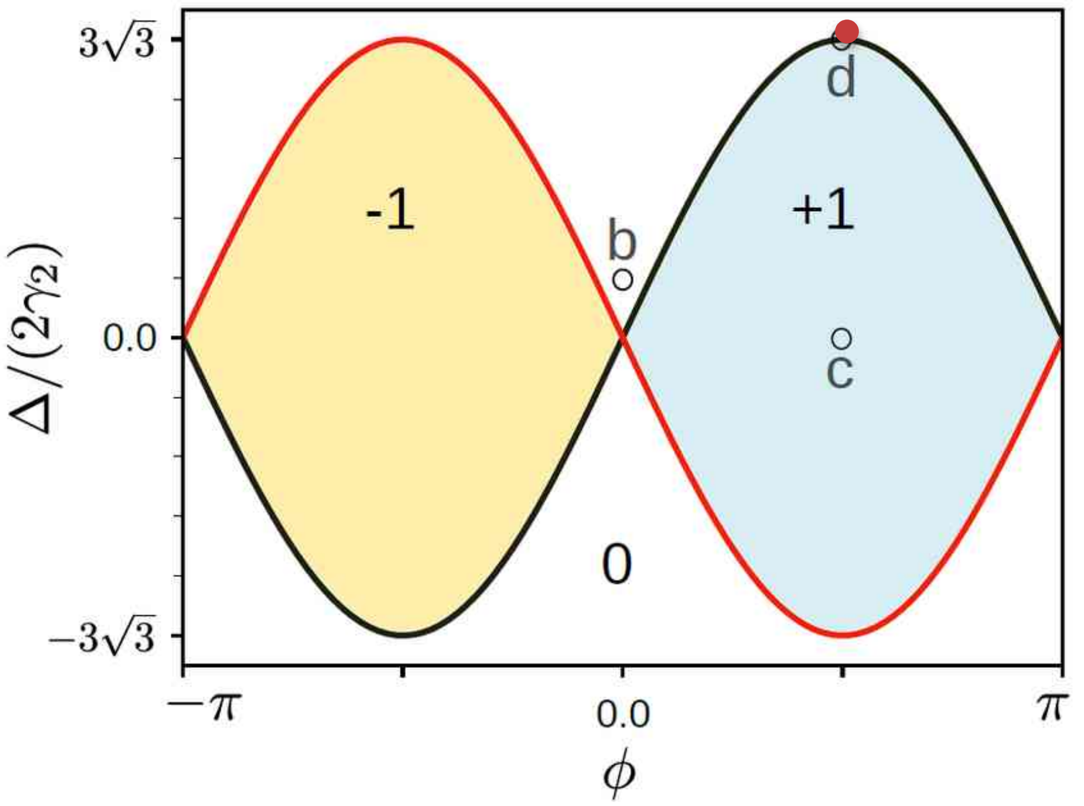

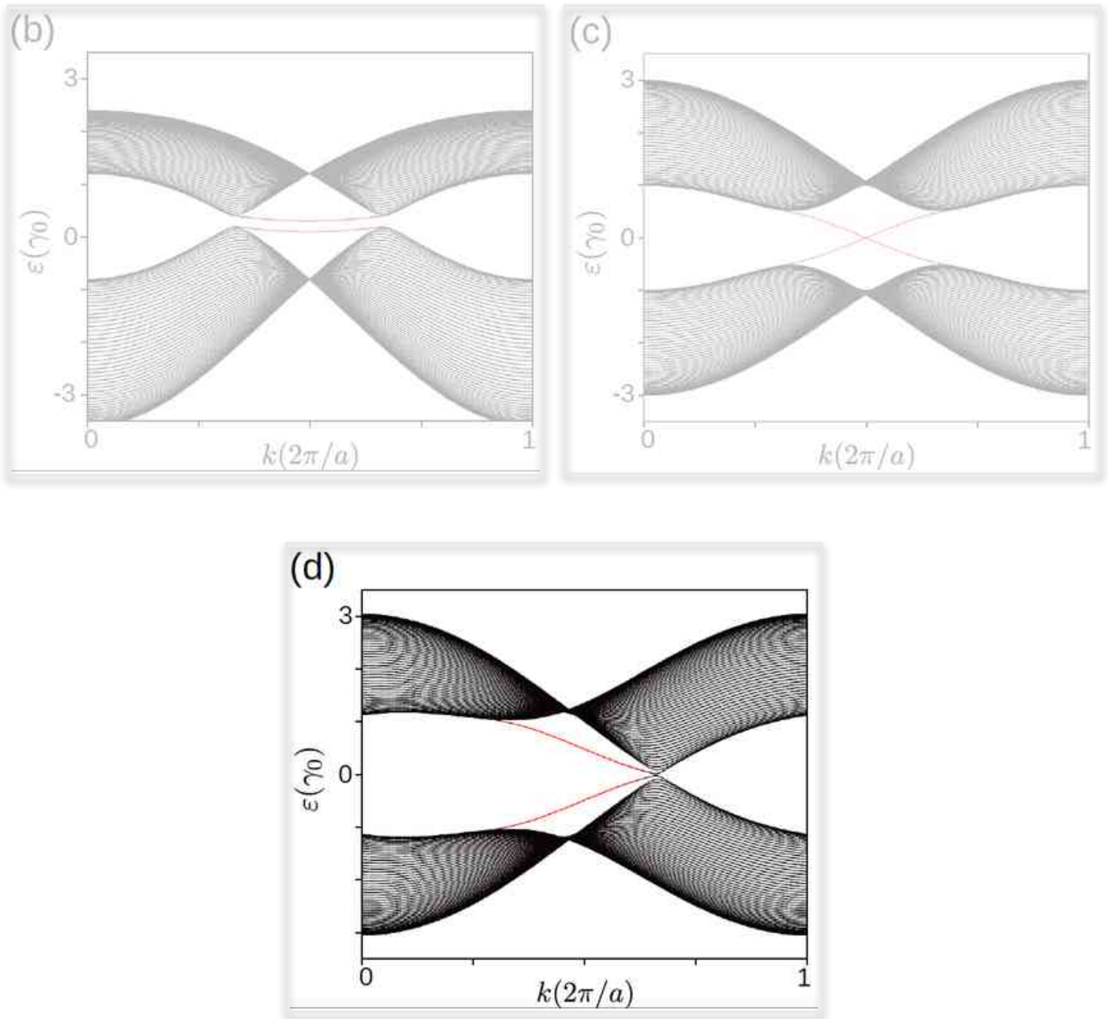

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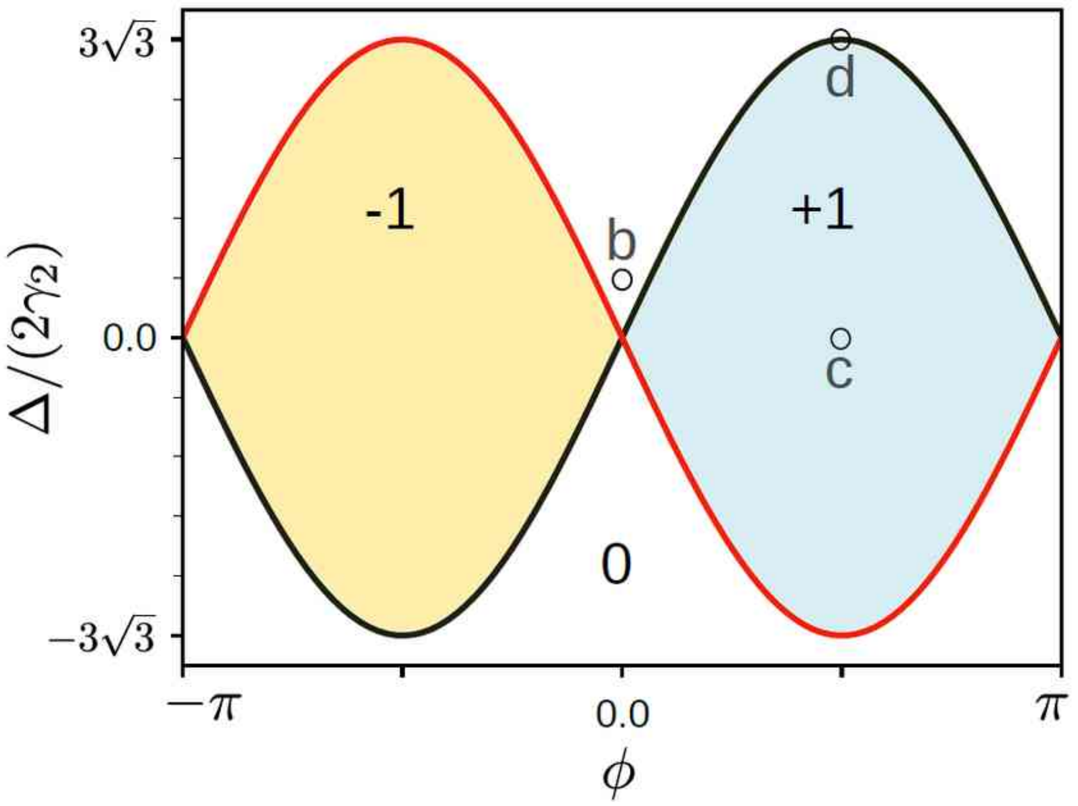
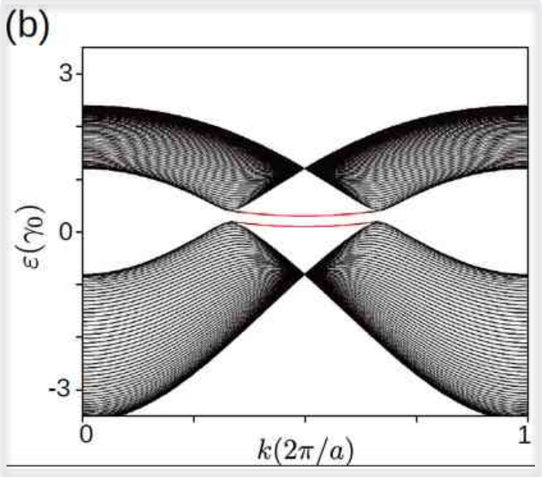
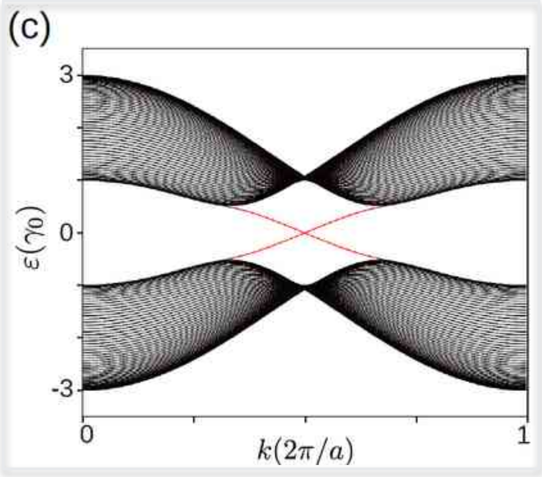
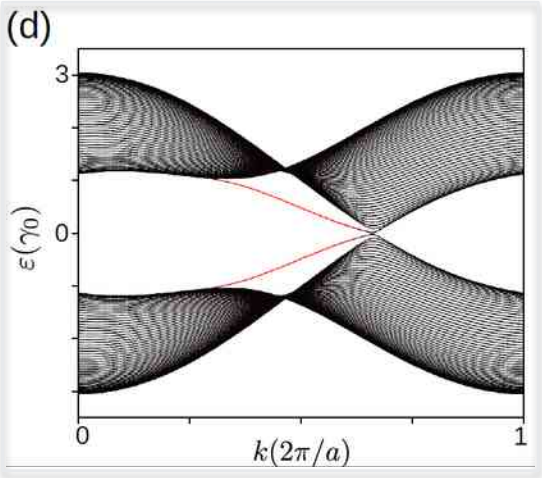








Summary — the Haldane model

- The Haldane model in the context of the search for quantum-Hall-like physics.
- Symmetries of graphene: inversion (IS) and time reversal (TRS).
- Opening a gap in graphene needs breaking IS or TRS.
- IS breaking: the Semenoff mass — a gap, but a trivial one.
- **TRS breaking:**
  - magnetic field → quantum Hall effect
  - Haldane term, a “local field” → a new phase: QH-like physics, but as a **lattice** property!

The Haldane phase, in the lab

LETTER

doi:10.1038/nature13915

Experimental realization of the topological Haldane model with ultracold fermions

Gregor Jotzu[1], Michael Messer[1], Rémi Desbuquois[1], Martin Lebrat[1], Thomas Uehlinger[1], Daniel Greif[1] & Tilman Esslinger[1]

Jotzu et al., Nature 515, 237 (2014) — ultracold fermions

Selected for a Viewpoint in *Physics*
PRL **114**, 187201 (2015) PHYSICAL REVIEW LETTERS week ending 8 MAY 2015

Precise Quantization of the Anomalous Hall Effect near Zero Magnetic Field

A. J. Bestwick,[1,2] E. J. Fox,[1,2] Xufeng Kou,[3] Lei Pan,[3] Kang L. Wang,[3] and D. Goldhaber-Gordon[1,2,*]
[1]*Department of Physics, Stanford University, Stanford, California 94305, USA*
[2]*Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[3]*Department of Electrical Engineering, University of California, Los Angeles, California 90095, USA*
(Received 16 January 2015; revised manuscript received 16 March 2015; published 4 May 2015)

We report a nearly ideal quantum anomalous Hall effect in a three-dimensional topological insulator thin film with ferromagnetic doping. Near zero applied magnetic field we measure exact quantization in the Hall resistance to within a part per 10 000 and a longitudinal resistivity under 1 Ω per square, with chiral edge transport explicitly confirmed by nonlocal measurements. Deviations from this behavior are found to be caused by thermally activated carriers, as indicated by an Arrhenius law temperature dependence. Using the deviations as a thermometer, we demonstrate an unexpected magnetocaloric effect and use it to reach near-perfect quantization by cooling the sample below the dilution refrigerator base temperature in a process approximating adiabatic demagnetization refrigeration.

DOI: 10.1103/PhysRevLett.114.187201 PACS numbers: 75.47.-m, 73.43.Fj, 75.45.+j, 75.50.Pp

Bestwick et al., Phys. Rev. Lett. 114, 187201 (2015) — quantum anomalous Hall effect

PART II

The Kane–Mele model

Quantum spin Hall effect in graphene

Quantum spin Hall effect in graphene

PRL **95,** 226801 (2005) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2005

Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele

Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

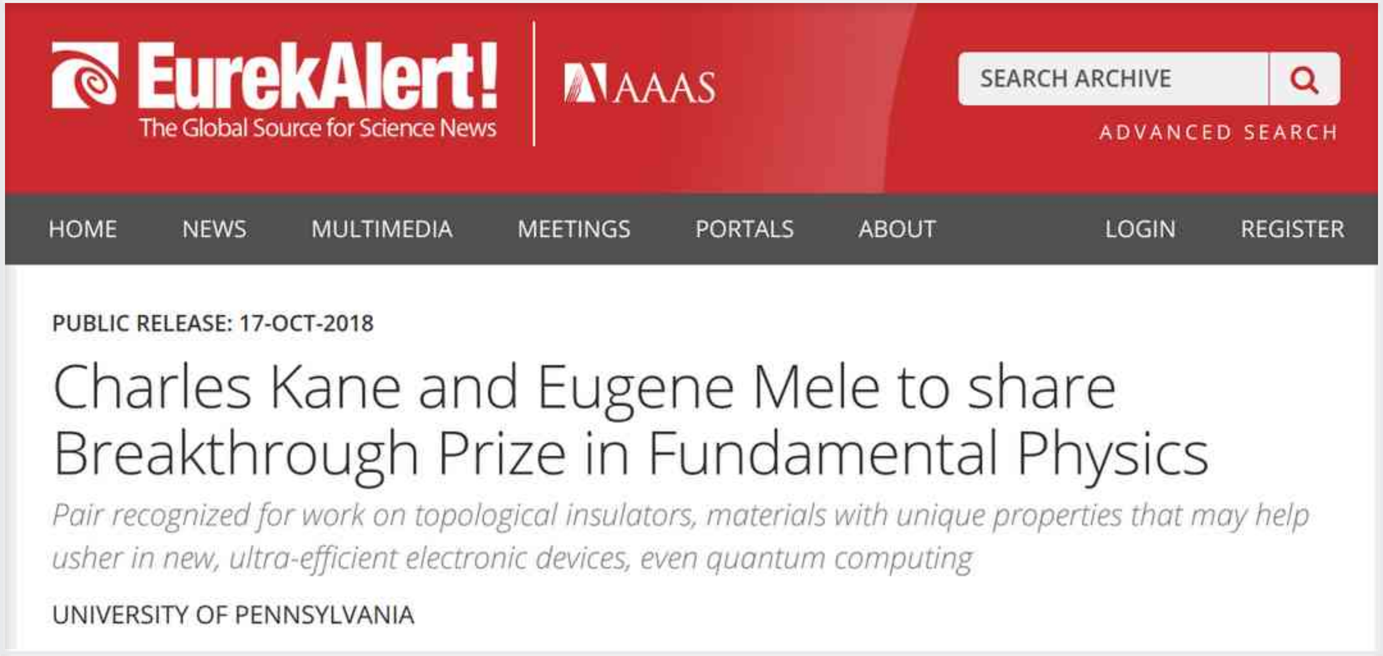
(Received 29 November 2004; published 23 November 2005)

We study the effects of spin orbit interactions on the low energy electronic structure of a single plane of graphene. We find that in an experimentally accessible low temperature regime the symmetry allowed spin orbit potential converts graphene from an ideal two-dimensional semimetallic state to a quantum spin Hall insulator. This novel electronic state of matter is gapped in the bulk and supports the transport of spin and charge in gapless edge states that propagate at the sample boundaries. The edge states are nonchiral, but they are insensitive to disorder because their directionality is correlated with spin. The spin and charge conductances in these edge states are calculated and the effects of temperature, chemical potential, Rashba coupling, disorder, and symmetry breaking fields are discussed.

DOI: 10.1103/PhysRevLett.95.226801 PACS numbers: 73.43.−f, 72.25.Hg, 73.61.Wp, 85.75.−d

C. L. Kane & E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005)

A groundbreaking prediction


EurekAlert! The Global Source for Science News | AAAS

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ADVANCED SEARCH

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PUBLIC RELEASE: 17-OCT-2018

Charles Kane and Eugene Mele to share Breakthrough Prize in Fundamental Physics

Pair recognized for work on topological insulators, materials with unique properties that may help usher in new, ultra-efficient electronic devices, even quantum computing

UNIVERSITY OF PENNSYLVANIA

C. L. Kane and E. J. Mele

They will share the $3 million award "for new ideas about topology and symmetry in physics, leading to the prediction of a new class of materials that conduct electricity only on their surface."

Context for a groundbreaking prediction

It's the end of 2004.

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666 22 OCTOBER 2004 VOL 306 SCIENCE www.sciencemag.org

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,[1] A. K. Geim,[1*] S. V. Morozov,[2] D. Jiang,[1] Y. Zhang,[1] S. V. Dubonos,[2] I. V. Grigorieva,[1] A. A. Firsov[2]

We describe monocrystalline graphitic films, which are a few atoms thick but are nonetheless stable under ambient conditions, metallic, and of remarkably high quality. The films are found to be a two-dimensional semimetal with a tiny overlap between valence and conductance bands, and they exhibit a strong ambipolar electric field effect such that electrons and holes in concentrations up to 10^{13} per square centimeter and with room-temperature mobilities of ~10,000 square centimeters per volt-second can be induced by applying gate voltage.

The ability to control electronic properties of a material by externally applied voltage is at the heart of modern electronics. In many cases, it is the electric field effect that allows one to vary the carrier concentration in a semiconductor device and, consequently, change an electric current through it. As the semiconductor industry is nearing the limits of performance improvements for the current technologies dominated by silicon, there is a constant search for new, nontraditional materials whose properties can be controlled by the electric field. The most notable recent examples of such materials are organic conductors (*1*) and carbon nanotubes (*2*). It has long been tempting to extend the use of the field effect to metals [e.g., to develop all-metallic transistors that could be scaled down to much smaller sizes and would consume less energy and operate at higher frequencies

[1]Department of Physics, University of Manchester, Manchester M13 9PL, UK. [2]Institute for Microelectronics Technology, 142432 Chernogolovka, Russia.

*To whom correspondence should be addressed. E-mail: geim@man.ac.uk

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Context for a groundbreaking prediction

It’s the end of 2004.

The playground is, once more, graphene.

This time, unlike at the time of Haldane’s model, graphene already exists in the lab — and there is a surge of interest in this newly discovered material.

Kane and Mele break through by proposing a new mechanism, one that transcends any particular material.

Which terms can open a gap?

Without spin, the only terms that can open a gap in the graphene lattice are:

[the idea: enumerate all possible terms that respect graphene's symmetries]

- a mass term breaking sublattice / inversion symmetry (e.g. graphene on BN)
- a mass term breaking TRS (e.g. the Haldane term)

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BUT the **spin–orbit interaction allows for a different kind of gap-opening term:**

Spin–orbit coupling

$$-\mathrm{i}\,\gamma_{SO} \sum_{\langle\langle i,j \rangle\rangle, s_z} \nu_{i,j}\, s_z \,|\, i, s_z \rangle \langle j, s_z \,|$$

± 1 depending on the spin

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[the idea: enumerate all possible terms that respect graphene's symmetries]

- a mass term breaking sublattice / inversion symmetry (e.g. graphene on BN)
- a mass term breaking TRS (e.g. the Haldane term)

BUT the **spin–orbit interaction allows for a different kind of gap-opening term:**

Spin–orbit coupling

$$-\mathrm{i}\,\gamma_{SO} \sum_{\langle\langle i,j \rangle\rangle, s_z} \nu_{i,j}\, s_z \,|\, i, s_z \rangle \langle j, s_z \,|$$

$\pm$ 1 depending on the spin

. Preserves Sz

. Preserves TRS

(this is: none of the symmetries of graphene are broken)

Aside: spin–orbit coupling

Relativistic correction to the Schrödinger equation. It can be thought of as a Zeeman effect for the electrons, caused by the (relative) motion of the nuclei.

It is more important for materials made of heavier atoms.

Typically it removes the orbital degeneracy.

Two copies of the Haldane model

BUT the **spin–orbit interaction allows for a different kind of gap-opening term:**

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Two copies of the Haldane Hamiltonian, one for each spin — with a reversed sign!

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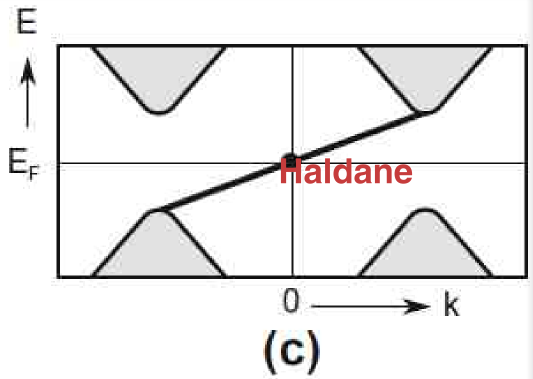

Two copies of the Haldane Hamiltonian, one for each spin — with a reversed sign!

Semi-infinite strip

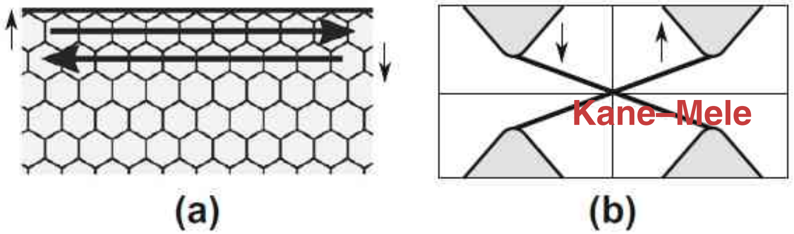

FIGURE 10 The edge states in graphene with the intrinsic spin orbit interaction have the spin filtered property that up and down spins propagate in opposite directions.

From: Topological Insulators, M. Franz & L. Molenkamp (Eds.), Elsevier (2013)

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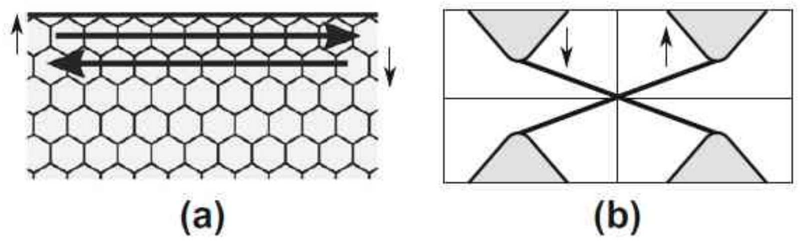

FIGURE 10 The edge states in graphene with the intrinsic spin orbit interaction have the spin filtered property that up and down spins propagate in opposite directions.

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PART II

Where does the robustness of the edge states come from?

Where does the robustness come from?

Preliminary question:

How robust are the edge states in the integer quantum Hall effect?

Where does the robustness come from?

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How robust are the edge states in the integer quantum Hall effect?

Hint: think about the topological phase in the SSH model — which condition did we have to impose to have a topological phase?

Kramers' degeneracy

The time-reversal operation ***T*** is anti-unitary:

$$\mathcal{T}^2 = \pm 1$$

The positive sign holds for spinless particles, the minus sign for spin-½ particles.
Therefore, in our case:

$$\mathcal{T}^2 = -1$$

One can then prove (and you are urged to do so as an exercise) that, given a time-reversal-invariant Hamiltonian,

$$[\,\mathcal{H}, \mathcal{T}\,] = 0$$

if ψ is a stationary state with energy E, then Tψ is also a linearly independent solution with the same eigenvalue.

Kramers' degeneracy — consequences

$$[\,\mathcal{H}, \mathcal{T}\,] = 0$$

if ψ is a stationary state with energy E, then Tψ is also a linearly independent solution with the same eigenvalue.

Consequences:

- The states at time-reversal-invariant momenta (k = 0 and k = π/a) are degenerate. One gets a sort of "1D Dirac point" at those momenta, protected by TRS.

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if ψ is a stationary state with energy E, then Tψ is also a linearly independent solution with the same eigenvalue.

Consequences:

- The states at time-reversal-invariant momenta ($k = 0$ and $k = \pi/a$) are degenerate. One gets a sort of "1D Dirac point" at those momenta, protected by TRS.

- A deeper connection between the time-reversed states k and $-k$: absence of backscattering (at the level of single-particle processes).

Proof: the matrix element of the Hamiltonian between two such time-reversed states vanishes, provided TRS is preserved.

No backscattering between Kramers partners

$$[\,\mathcal{H},\mathcal{T}\,] = 0$$

take the time-reversed partner ψ = Tφ and compute:

$$\langle\psi|H\varphi\rangle = \langle\mathcal{T}\varphi|H\varphi\rangle = \langle\mathcal{T}H\varphi|\mathcal{T}^2\varphi\rangle$$

$$= -\langle\mathcal{T}H\varphi|\varphi\rangle \qquad \text{using } \mathcal{T}^2 = -1$$

$$= -\langle H\mathcal{T}\varphi|\varphi\rangle \qquad \text{using } [\mathcal{T},H] = 0$$

$$= -\langle\mathcal{T}\varphi|H|\varphi\rangle$$

J. Cayssol, arXiv:1303.5902

Intrinsic vs Rashba spin–orbit coupling

Discussion on the ISO and Rashba terms.

Band inversion

Discussion on band inversion — the effect of the Semenoff versus the Haldane term.

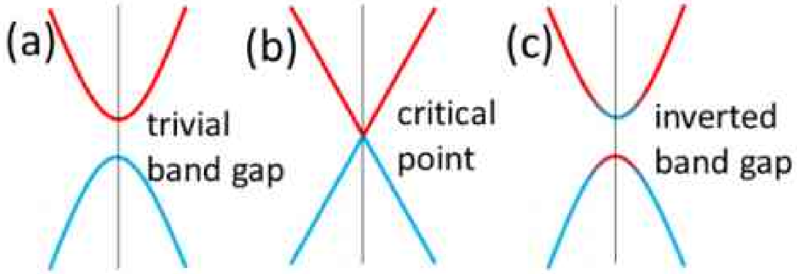


Fig. 1. Schematic band inversion between two bands derived from the natural order of atomic levels. The trivial band gap in (a) closes at a critical point in (b), and reopens inverted in (c) with the two states swapping their orbital characters at the symmetry point.

From: arXiv:1603.03576

PART II

Realization of topological insulators

The era of topological insulators

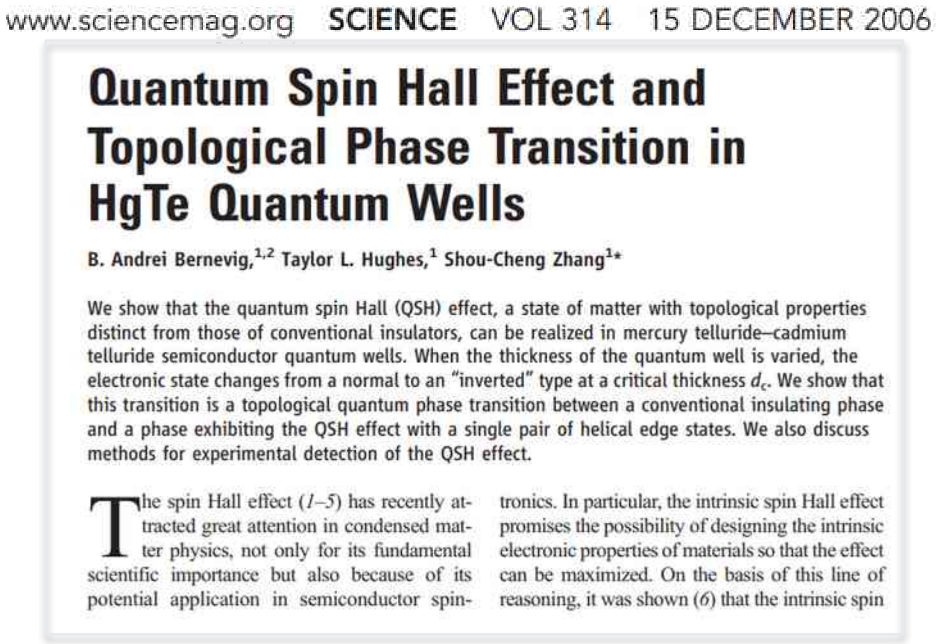
Spin–orbit coupling in graphene turned out to be too small to lead to a sizeable effect (gap ~ mK).

The era of topological insulators

Spin–orbit coupling in graphene turned out to be too small to lead to a sizeable effect (gap ~ mK).

But the story was not over — because they were right about the mechanism.

Bernevig, Hughes & Zhang, Science 314, 1757 (2006)



www.sciencemag.org SCIENCE VOL 314 15 DECEMBER 2006

Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

B. Andrei Bernevig,[1,2] Taylor L. Hughes,[1] Shou-Cheng Zhang[1*]

We show that the quantum spin Hall (QSH) effect, a state of matter with topological properties distinct from those of conventional insulators, can be realized in mercury telluride–cadmium telluride semiconductor quantum wells. When the thickness of the quantum well is varied, the electronic state changes from a normal to an "inverted" type at a critical thickness d_c. We show that this transition is a topological quantum phase transition between a conventional insulating phase and a phase exhibiting the QSH effect with a single pair of helical edge states. We also discuss methods for experimental detection of the QSH effect.

The spin Hall effect (*1*–*5*) has recently attracted great attention in condensed matter physics, not only for its fundamental scientific importance but also because of its potential application in semiconductor spintronics. In particular, the intrinsic spin Hall effect promises the possibility of designing the intrinsic electronic properties of materials so that the effect can be maximized. On the basis of this line of reasoning, it was shown (*6*) that the intrinsic spin

The era of topological insulators

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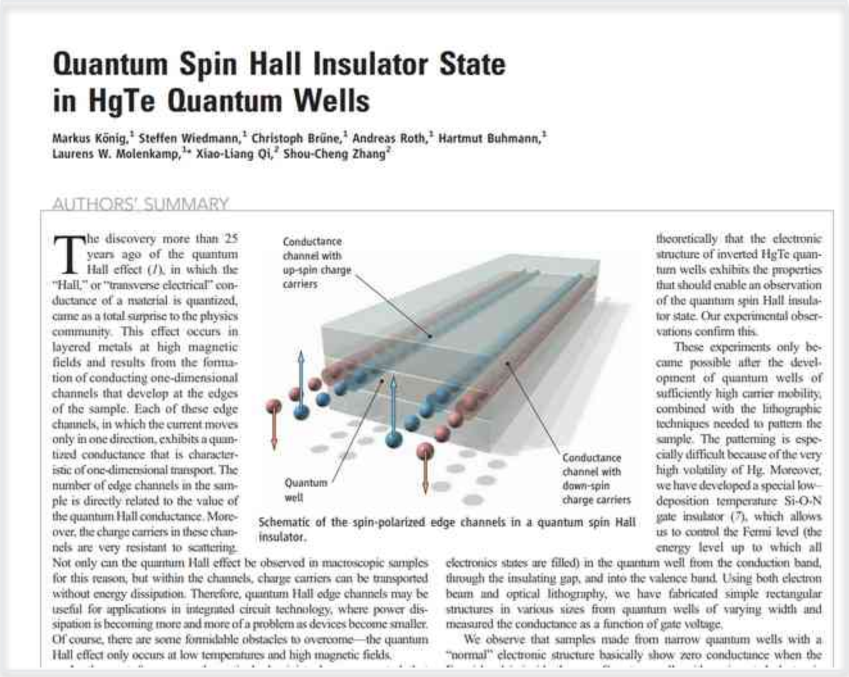
König et al., Science 318, 766 (2007)



Quantum Spin Hall Insulator State in HgTe Quantum Wells

Markus König,[1] Steffen Wiedmann,[1] Christoph Brüne,[1] Andreas Roth,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1*] Xiao-Liang Qi,[2] Shou-Cheng Zhang[2]

AUTHORS' SUMMARY

The discovery more than 25 years ago of the quantum Hall effect (1), in which the "Hall," or "transverse electrical" conductance of a material is quantized, came as a total surprise to the physics community. This effect occurs in layered metals at high magnetic fields and results from the formation of conducting one-dimensional channels that develop at the edges of the sample. Each of these edge channels, in which the current moves only in one direction, exhibits a quantized conductance that is characteristic of one-dimensional transport. The number of edge channels in the sample is directly related to the value of the quantum Hall conductance. Moreover, the charge carriers in these channels are very resistant to scattering. Not only can the quantum Hall effect be observed in macroscopic samples for this reason, but within the channels, charge carriers can be transported without energy dissipation. Therefore, quantum Hall edge channels may be useful for applications in integrated circuit technology, where power dissipation is becoming more and more of a problem as devices become smaller. Of course, there are some formidable obstacles to overcome—the quantum Hall effect only occurs at low temperatures and high magnetic fields.

Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

theoretically that the electronic structure of inverted HgTe quantum wells exhibits the properties that should enable an observation of the quantum spin Hall insulator state. Our experimental observations confirm this.

These experiments only became possible after the development of quantum wells of sufficiently high carrier mobility, combined with the lithographic techniques needed to pattern the sample. The patterning is especially difficult because of the very high volatility of Hg. Moreover, we have developed a special low–deposition temperature Si-O-N gate insulator (7), which allows us to control the Fermi level (the energy level up to which all electronics states are filled) in the quantum well from the conduction band, through the insulating gap, and into the valence band. Using both electron beam and optical lithography, we have fabricated simple rectangular structures in various sizes from quantum wells of varying width and measured the conductance as a function of gate voltage.

We observe that samples made from narrow quantum wells with a "normal" electronic structure basically show zero conductance when the

The era of topological insulators

Spin–orbit coupling in graphene turned out to be too small to lead to a sizeable effect (gap ~ mK).

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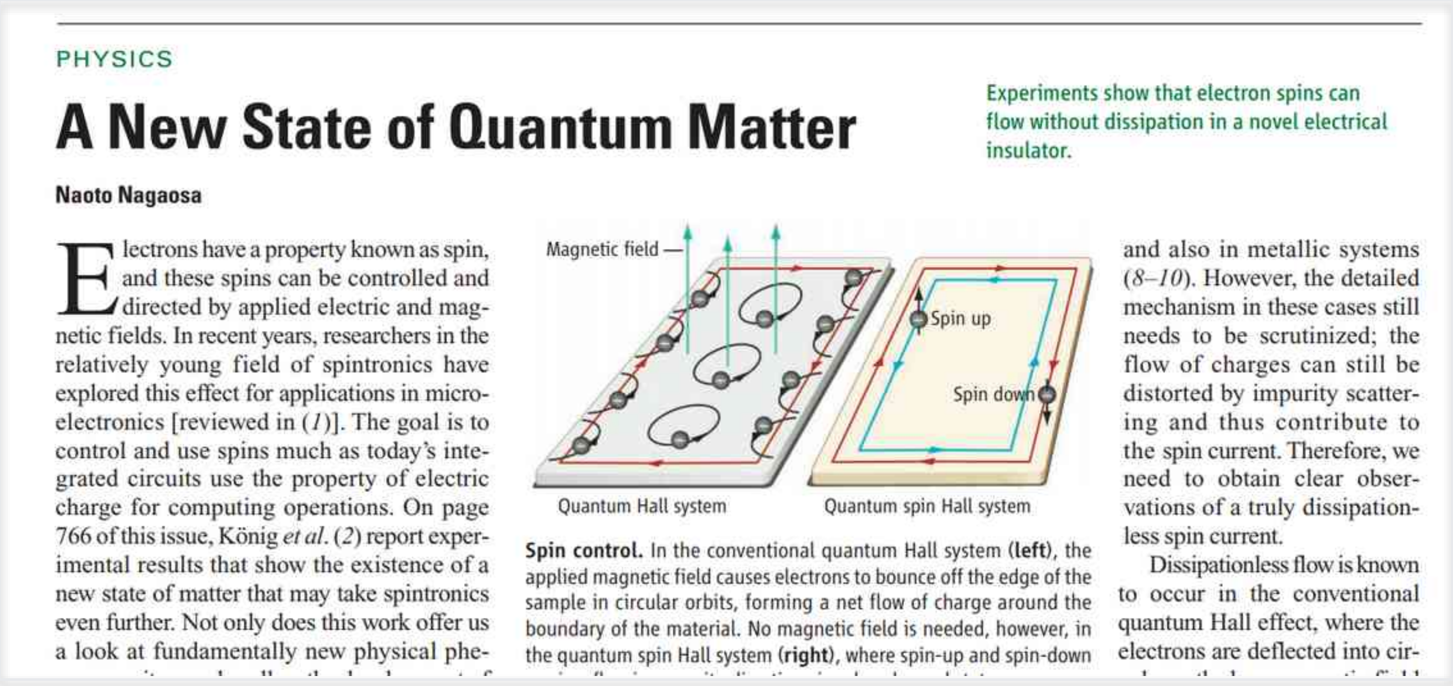

PHYSICS

A New State of Quantum Matter

Experiments show that electron spins can flow without dissipation in a novel electrical insulator.

Naoto Nagaosa

Electrons have a property known as spin, and these spins can be controlled and directed by applied electric and magnetic fields. In recent years, researchers in the relatively young field of spintronics have explored this effect for applications in microelectronics [reviewed in (*1*)]. The goal is to control and use spins much as today's integrated circuits use the property of electric charge for computing operations. On page 766 of this issue, König *et al.* (*2*) report experimental results that show the existence of a new state of matter that may take spintronics even further. Not only does this work offer us a look at fundamentally new physical phe-

Spin control. In the conventional quantum Hall system (**left**), the applied magnetic field causes electrons to bounce off the edge of the sample in circular orbits, forming a net flow of charge around the boundary of the material. No magnetic field is needed, however, in the quantum spin Hall system (**right**), where spin-up and spin-down

and also in metallic systems (*8*–*10*). However, the detailed mechanism in these cases still needs to be scrutinized; the flow of charges can still be distorted by impurity scattering and thus contribute to the spin current. Therefore, we need to obtain clear observations of a truly dissipationless spin current.

Dissipationless flow is known to occur in the conventional quantum Hall effect, where the electrons are deflected into cir-

N. Nagaosa, Science 318, 758 (2007) — perspective on the König et al. experiment

Summary — the Kane–Mele model

- Spin–orbit coupling provides a different, time-reversal-invariant, mass term.
- This term — different from Haldane's or Semenoff's — allows for a physical mechanism permitting helical edge states.
- At the core, this is just two time-reversed copies of the Haldane Hamiltonian.
- **The interesting point: there is topological protection as long as TRS is preserved.**
- In this sense, the topological states of the Kane–Mele Hamiltonian are weaker than those of the Haldane model.

Faraway, yet so close!

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

geometrical constant of order unity, and g is the Landé g factor for the electrons.

While the particular model presented here is unlikely to be directly physically realizable, it indicates that, at least in principle, the QHE can be placed in the wider context of phenomena associated with broken time-reversal invariance, and does not necessarily require external magnetic fields, but could occur as a consequence of magnetic ordering in a quasi-two-dimensional system.

This requirement is *not* fulfilled by the physical system (a domain wall in a PbTe-type semiconductor) in which Fradkin, Dagotto, and Boyanovsky[8] (FDB) have recently proposed related effects may be realized. In this model, spin-orbit coupling is supposed to give rise to the effect, but this does *not* break time-reversal symmetry. In fact, in "simplifying" the p bands of the Hamiltonian that describes PbTe, FDB introduce an unphysical effective spin-dependent hopping term that is *odd* under time reversal, and thus break the time-reversal invariance of the original physically motivated model. This, rather than any topological character of the domain wall, is the reason that FDB find the "parity anomaly" at the end of their calculation.

I thank E. Fradkin and T. A. L. Ziman for very useful discussions. The author would like to thank the Alfred P. Sloan Foundation for financial support.

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[2]R. Jackiw, Phys. Rev. D **27**, 2375 (1984).
[3]H. B. Nielsen and M. Ninomiya, Nucl. Phys. **B185**, 20 (1981), and **B193**, 173 (1981).
[4]D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).
[5]G. Semenoff, Phys. Rev. Lett. **53**, 2449 (1984).
[6]The full group of the wave vector at zone corners contains additional *antiunitary* operations, but these do not give rise to extra degeneracies.
[7]P. Středa, J. Phys. C **15**, L717 (1982); A. Widom, Phys. Lett. **90A**, 474 (1982).
[8]E. Fradkin, E. Dagotto, and D. Boyanovsky, Phys. Rev. Lett. **57**, 2967 (1986), and **58**, 961(E) (1987); D. Boyanovsky, E. Dagotto, and E. Fradkin, Nucl. Phys. **B285**, 340 (1987).

The last page of Haldane's 1988 paper

Questions to take home

Questions:

. What happens with the Chern number in this time-reversal-invariant situation?

Hint: remember the two golden rules for the Berry curvature.

. What happens with the Hall conductance?

PART III

Beyond equilibrium and beyond Hermiticity

A short outlook: driven and non-Hermitian topological phases

Periodic driving: phases with no static analogue

A periodically driven (Floquet) system has quasi-energies defined only modulo $\hbar\Omega$ — already a qualitatively new ingredient.

The anomalous Floquet insulator: every bulk Chern number can vanish, and yet the system hosts robust chiral edge states — a phase with no static analogue.

One realization: shining circularly polarized light on graphene turns it into a tunable Floquet topological insulator, replaying much of the Haldane-model phenomenology of Part I — but driven by light instead of a static flux.

M. S. Rudner, N. H. Lindner, E. Berg & M. Levin, Phys. Rev. X 3, 031005 (2013) · P. M. Perez-Piskunow, G. Usaj, C. A. Balseiro & L. E. F. Foà Torres, Phys. Rev. B 89, 121401(R) (2014)

And the story continues:
more experiments, this time with continuous lasers

nature communications

Article

https://doi.org/10.1038/s41467-025-57335-2

Signatures of Floquet electronic steady states in graphene under continuous-wave mid-infrared irradiation

Received: 14 February 2024

Accepted: 19 February 2025

Published online: 28 February 2025

Check for updates

Yijing Liu[1,9], Christopher Yang[2,9], Gabriel Gaertner[3], John Huckabee[3], Alexey V. Suslov[4], Gil Refael[2], Frederik Nathan[5], Cyprian Lewandowski[4,6], Luis E. F. Foa Torres[7], Iliya Esin[2,8], Paola Barbara[1] & Nikolai G. Kalugin[3]

Article

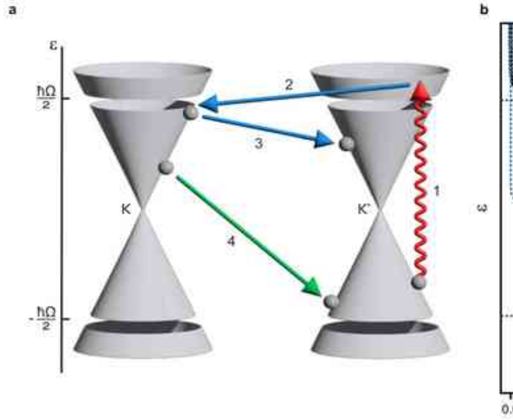

Fig. 2 | Expected electronic steady-state distribution. a Key scattering processes between the graphene K and K' valleys facilitated by photons (arrow 1), surface acoustic phonons (arrows 2,3), and graphene acoustic phonons (arrow 4) that contribute to the steady state. **b** Steady state distribution (black solid curve) for $P = 1.9$ mW/μm² and equilibrium distribution (blue dashed curve) of electrons at doping $E_F = 0$, where the longitudinal photoconductivity is enhanced relative to equilibrium. Here, ε denotes the quasienergy and $F_{k\alpha}$ denotes the electronic occupation. **c** Steady state distribution (black solid curve) of electrons at doping $E_F = 0.35\,\hbar\Omega$ and $P = 0.8$ mW/μm², where the photoconductivity is suppressed

And the story continues:
direct observation in graphene achieved at last in 2025

nature physics

Article https://doi.org/10.1038/s41567-025-02888-8

Observation of Floquet–Bloch states in monolayer graphene

Received: 30 September 2024

Accepted: 21 March 2025

Published online: 01 May 2025

Dongsung Choi[1,8], Masataka Mogi[2,3,8], Umberto De Giovannini[4,5], Doron Azoury[2], Baiqing Lv[2,6], Yifan Su[2], Hannes Hübener[4], Angel Rubio[4,7] & Nuh Gedik[2]

nature physics

Article https://doi.org/10.1038/s41567-025-02889-7

Observation of Floquet states in graphene

Received: 30 September 2024

Accepted: 21 March 2025

Published online: 6 May 2025

Marco Merboldt[1,9], Michael Schüler[2,3,9], David Schmitt[1], Jan Philipp Bange[1], Wiebke Bennecke[1], Karun Gadge[4], Klaus Pierz[5], Hans Werner Schumacher[5], Davood Momeni[5], Daniel Steil[1], Salvatore R. Manmana[4], Michael A. Sentef[6,7], Marcel Reutzel[1] & Stefan Mathias[1,8]

And the story continues:
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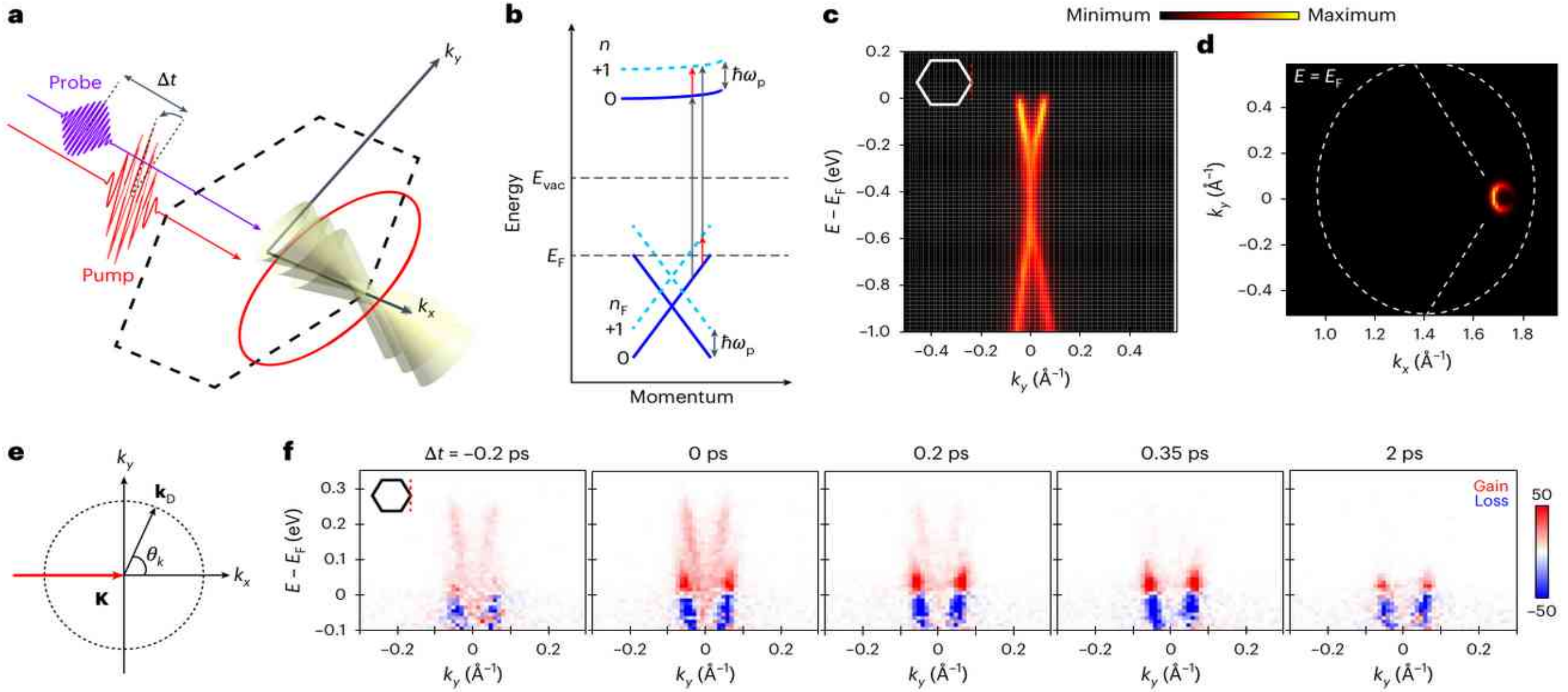

Non-Hermitian systems: Two challenges for a topological classification

When a system exchanges energy with its environment — gain and loss, a finite quasiparticle lifetime, coupling to a continuum — its effective Hamiltonian becomes non-Hermitian: $H \neq H^\dagger$, and the spectrum is complex.

. Defining a gap for a complex spectrum.

There is no order relation for complex numbers.

. Extreme sensitivity of the spectrum to the boundary conditions.

The spectra of the finite and infinite system may not match, no matter how large — one might be gapped while the other is not.

L. E. F. Foà Torres, J. Phys. Mater. 3, 014002 (2020) · V. M. Martinez Alvarez, J. E. Barrios Vargas, M. Berdakin & L. E. F. Foà Torres, Eur. Phys. J. Spec. Top. 227, 1295 (2018)

Two challenges for a topological classification

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An ordered non-Hermitian lattice may have every eigenstate anomalously localized at the boundary — the non-Hermitian skin effect.

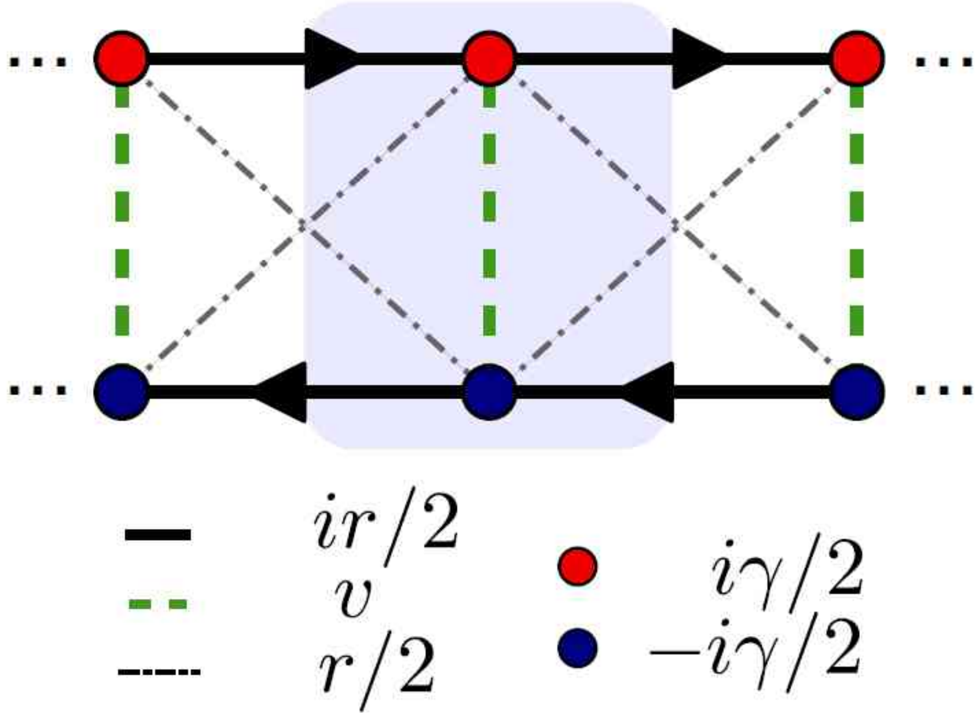
L. E. F. Foà Torres, J. Phys. Mater. 3, 014002 (2020) · V. M. Martinez Alvarez, J. E. Barrios Vargas, M. Berdakin & L. E. F. Foà Torres, Eur. Phys. J. Spec. Top. 227, 1295 (2018)

A usual 1d system — one new ingredient

T. E. Lee, PRL **116**, 133903 (2016)

$$H_k = h_x\sigma_x + \left(h_z + \frac{i\gamma}{2}\right)\sigma_z,$$

$$h_x = v + r\cos k, \qquad h_z = r\sin k,$$



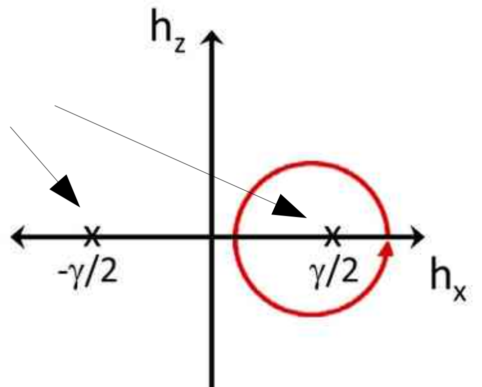
Chiral symmetry
(same as SSH model)

PT symmetry
(P: parity, T: Time-reversal symmetry)

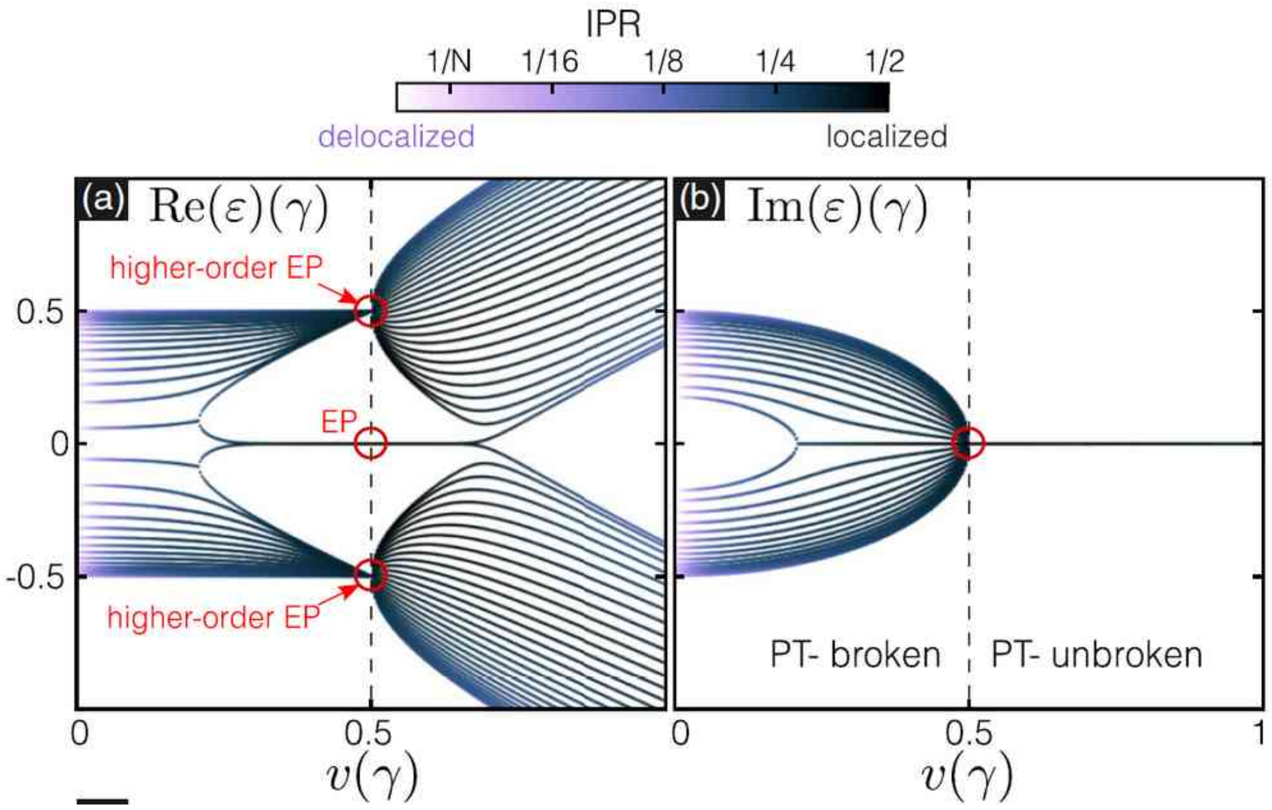
$$\mathcal{P}\mathcal{T}H\mathcal{T}^{-1}\mathcal{P}^{-1} = H$$

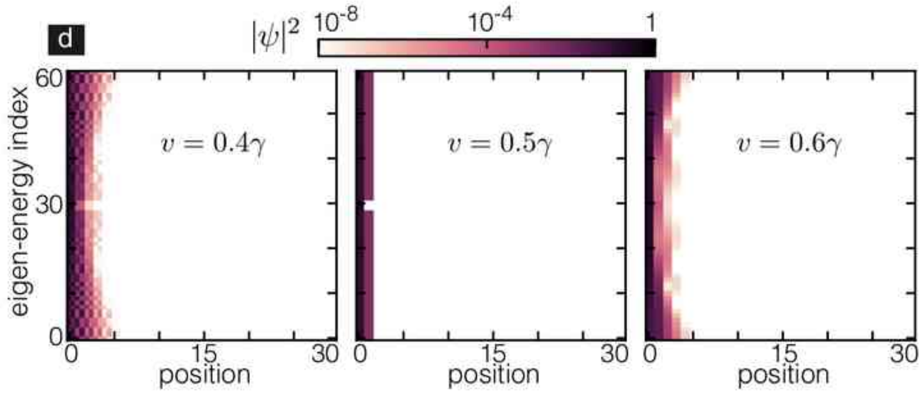
(the eigenvalues of H can be real, despite the non-Hermiticity)

exceptional points



All the (bulk) states condense at the boundary





At v = w = 0.5γ every one of the 60 eigenstates coalesces onto the same edge cell: a pristine, ordered chain, devoid of bulk states.

V. M. Martinez Alvarez, J. E. Barrios Vargas & L. E. F. Foà Torres, Phys. Rev. B 97, 121401(R) (2018)

Not disorder — coalescence

Anderson localization needs interference and disorder. Here a pristine, ordered lattice localizes every state by sheer coalescence of eigenvectors at exceptional points — the Hamiltonian becomes defective (non-diagonalizable).

An “illusion of choice”: formally many eigenstates, but only a handful of independent directions survive in practice.

“An Aleph is one of the points in space that contains all other points.” — J. L. Borges, El Aleph. At a higher-order exceptional point, a number of states scaling with the system size coalesce — a physical Aleph.

This condensation can be read as an environment-mediated interaction: the non-Hermitian part of H mixes the eigenstates of its Hermitian half until they align onto a few selected directions.

Anomalous localization reported

PHYSICAL REVIEW B **97**, 121401(R) (2018)

Rapid Communications

Non-Hermitian robust edge states in one dimension: Anomalous localization and eigenspace condensation at exceptional points

V. M. Martinez Alvarez,[1] J. E. Barrios Vargas,[2] and L. E. F. Foa Torres[2,*]

[1]*Departamento de Física, Laboratório de Física Teórica e Computacional, Universidade Federal de Pernambuco, Recife 50670-901, Pernambuco, Brazil*

[2]*Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Santiago 837.0415, Chile*

(Received 25 August 2017; revised manuscript received 17 February 2018; published 7 March 2018)

Capital to topological insulators, the bulk-boundary correspondence ties a topological invariant computed from the bulk (extended) states with those at the boundary, which are hence robust to disorder. Here we put forward a different ordering unique to non-Hermitian lattices whereby a pristine system becomes devoid of extended states, a property which turns out to be robust to disorder. This is enabled by a peculiar type of non-Hermitian degeneracy where a macroscopic fraction of the states coalesce at a single point with a geometrical multiplicity of 1.

DOI: 10.1103/PhysRevB.97.121401

Non-hermitian skin effect

PHYSICAL REVIEW LETTERS **121,** 086803 (2018)

Editors' Suggestion

Edge States and Topological Invariants of Non-Hermitian Systems

Shunyu Yao[1] and Zhong Wang[1,2,*]

[1]*Institute for Advanced Study, Tsinghua University, Beijing 100084, China*

[2]*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

(Received 6 March 2018; published 21 August 2018)

The bulk-boundary correspondence is among the central issues of non-Hermitian topological states. We show that a previously overlooked "non-Hermitian skin effect" necessitates redefinition of topological invariants in a generalized Brillouin zone. The resultant phase diagrams dramatically differ from the usual Bloch theory. Specifically, we obtain the phase diagram of the non-Hermitian Su-Schrieffer-Heeger model, whose topological zero modes are determir
Hamiltonian-based topological number. Our
boundary correspondence and introduces tl

DOI: 10.1103/PhysRevLett.121.086803

Affirmative answers are obtained in this Letter.

We start from solving a 1D model. Interestingly, all the eigenstates of an open chain are found to be localized near the boundary (dubbed the "non-Hermitian skin effect"), in contrast to the extended Bloch waves in Hermitian cases. In the simplest situations, this effect can be understood in

And the story goes on:
The topological origin of the non-Hermitian skin effect

PHYSICAL REVIEW LETTERS **125,** 126402 (2020)

Editors' Suggestion

Correspondence between Winding Numbers and Skin Modes in Non-Hermitian Systems

Kai Zhang,[1,2,*] Zhesen Yang,[1,2,*] and Chen Fang[1,3,4,†]
[1]*Beijing National Laboratory for Condensed Matter Physics, and Institute of Physics,*
f Sciences, Beijing 100190, China
demy of Sciences, Beijing 100049, China
ratory, Dongguan, Guangdong 523808, China
gical Quantum Computation, Beijing 100190, China

ted 24 August 2020; published 15 September 2020)

winding of "energy" (eigenvalue of Hamiltonian) on the
Brillouin zone with periodic boundary condition, and the
ry conditions in non-Hermitian systems. We show that the

PHYSICAL REVIEW LETTERS **124,** 086801 (2020)

Topological Origin of Non-Hermitian Skin Effects

Nobuyuki Okuma,[1,*] Kohei Kawabata,[2] Ken Shiozaki,[1] and Masatoshi Sato[1]
[1]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[2]*Department of Physics, University of Tokyo, 7-3-1 Hongo*

(Received 7 October 2019; accepted 29 January 2

A unique feature of non-Hermitian systems is the skin effe
boundary conditions. Here, we reveal that the skin effect origina
Such a topological origin not merely explains the universal featu
to new types of the skin effects—symmetry-protected skin effe
effect protected by time-reversal symmetry. On the basis of t
possible other skin effects in arbitrary dimensions. Our work p
bulk-boundary correspondence and the skin effects in non-He

DOI: 10.1103/PhysRevLett.124.086801

PHYSICAL REVIEW LETTERS **124,** 056802 (2020)

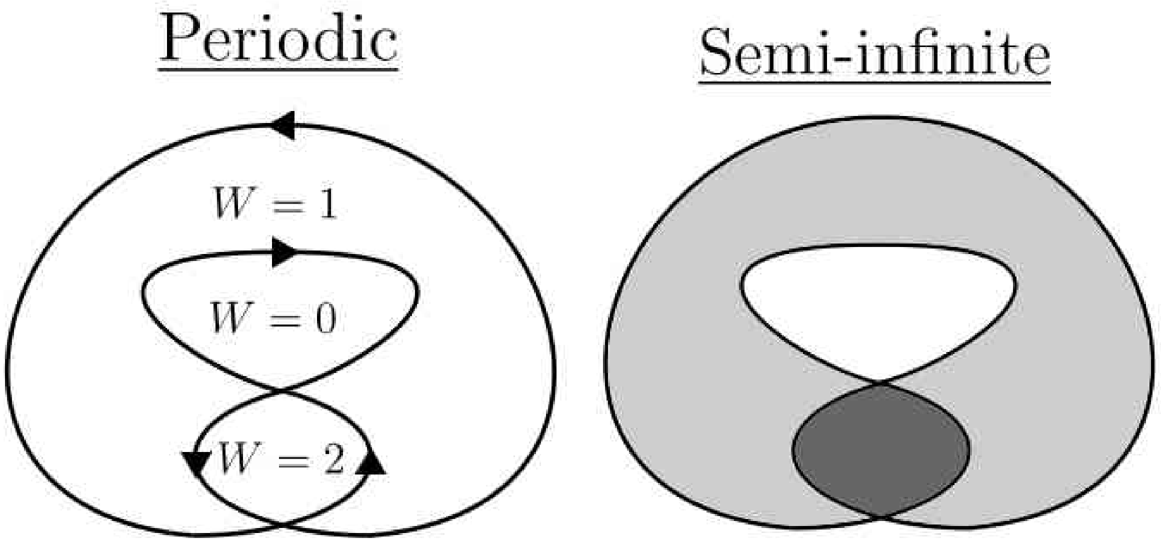
Non-Hermitian Boundary Modes and Topology

Dan S. Borgnia, Alex Jura Kruchkov, and Robert-Jan Slager
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 3 May 2019; revised manuscript received 25 November 2019; accepted 17 January 2020; published 7 February 2020)

The skin effect originates from intrinsic non-Hermitian spectral winding topology of the Bloch Hamiltonian.

$$W(E) := \int_0^{2\pi} \frac{dk}{2\pi i} \frac{d}{dk} \log \det (H(k) - E)$$



The skin effect originates from intrinsic non-Hermitian spectral winding topology of the Bloch Hamiltonian.

$$\tilde{H}(k, E_r) := \begin{pmatrix} 0 & H(k) - E_r \\ H^\dagger(k) - E_r^* & 0 \end{pmatrix}$$

FRONTIERS OF PHYSICS

REVIEW ARTICLE
Volume 18 / Issue 5 / 53605 / 2023

Topological non-Hermitian skin effect

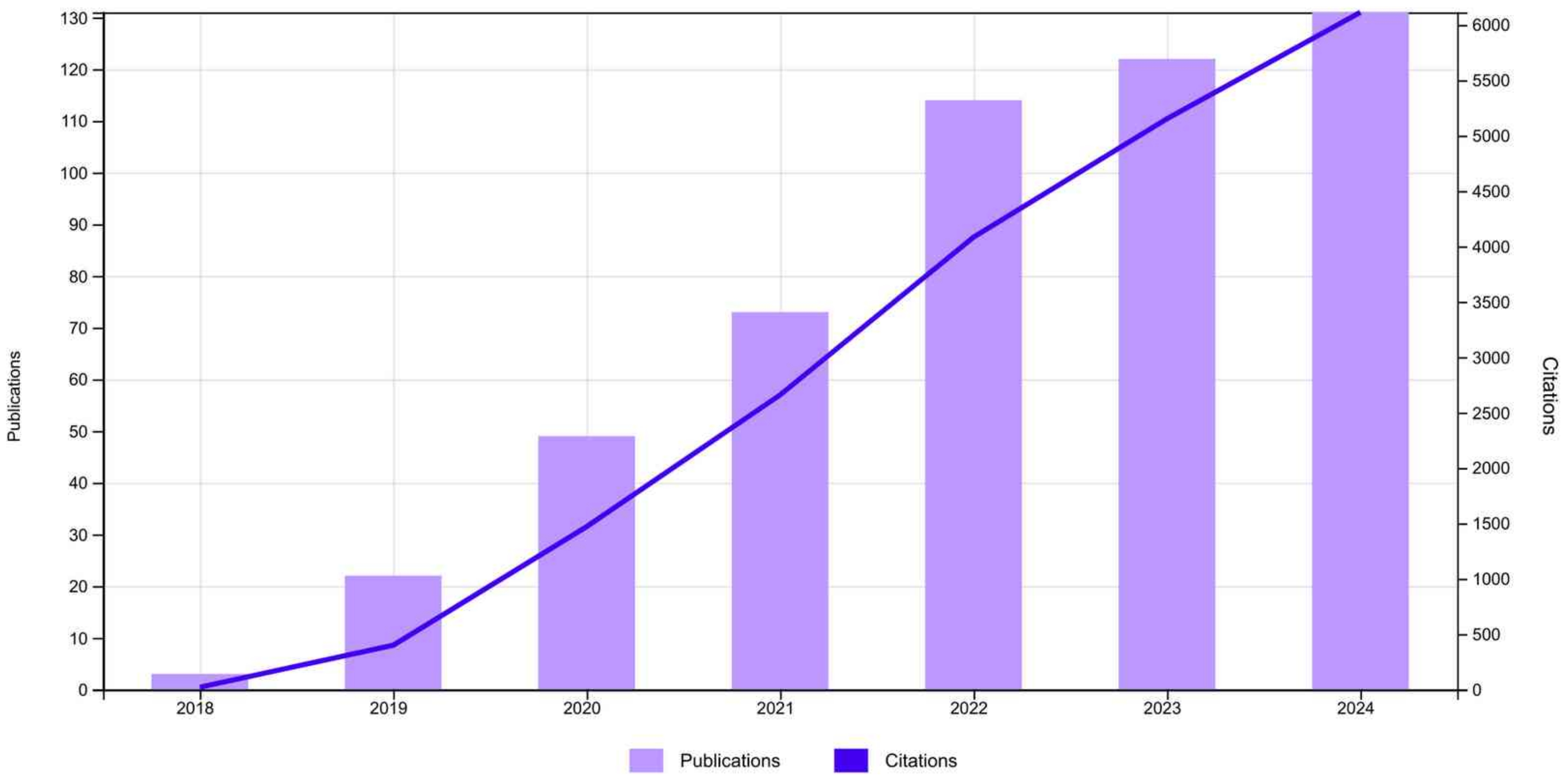
Rijia Lin[1,*], Tommy Tai[2,3,*,†], Linhu Li[1,‡], Ching Hua Lee[3,§]

1 Guangdong Provincial Key Laboratory of Quantum Metrology and Sensing & Schoo
Sun Yat-Sen University (Zhuhai Campus), Zhuhai 519082, China
2 Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 021
3 Department of Physics, National University of Singapore, Singapore 117542, Singap
Corresponding authors. E-mail: †tommytai@mit.edu, ‡lilh56@mail.sysu.edu.cn, §phyl
*These authors contributed equally to this work
Received April 8, 2023; accepted May 8, 2023

closed to a topological phase transition. Another interpretation is to consider the evolution of the system from PBCs to OBCs, which changes the topological structure of the spectrum by passing through a series of EPs [44], but it does not provides a solution to restore the BBC either. This problem is eventually solved in 2018 by the sensational formulation of the non-Hermitian skin effect (NHSE), under which all eigenstates are spatially localized at the boundary of a system under the OBCs [45, 46]. Independently discovered by two research groups [45, 46], the NHSE has motivated the so-called non-Bloch band theory, recovering a generalized BBC for non-Hermitian Hamiltonians [46–49]. A parallel formulation of the non-Bloch Hamiltonian is to consider the PBC-OBC spectral evolution with a tunable hopping strength

45. V. M. Martinez Alvarez, J. E. Barrios Vargas, and L. E. F. Foa Torres, Non-Hermitian robust edge states in one dimension: Anomalous localization and eigenspace condensation at exceptional points, *Phys. Rev. B* 97(12), 121401 (2018)
46. S. Yao and Z. Wang, Edge states and topological invariants of non-Hermitian systems, *Phys. Rev. Lett.* 121(8), 086803 (2018)

Number of publications containing “non-hermitian skin effect” in the title, or abstract, until 11Nov2024 (source: Web of Science)



Anomalous localization/non-Hermitian skin effect

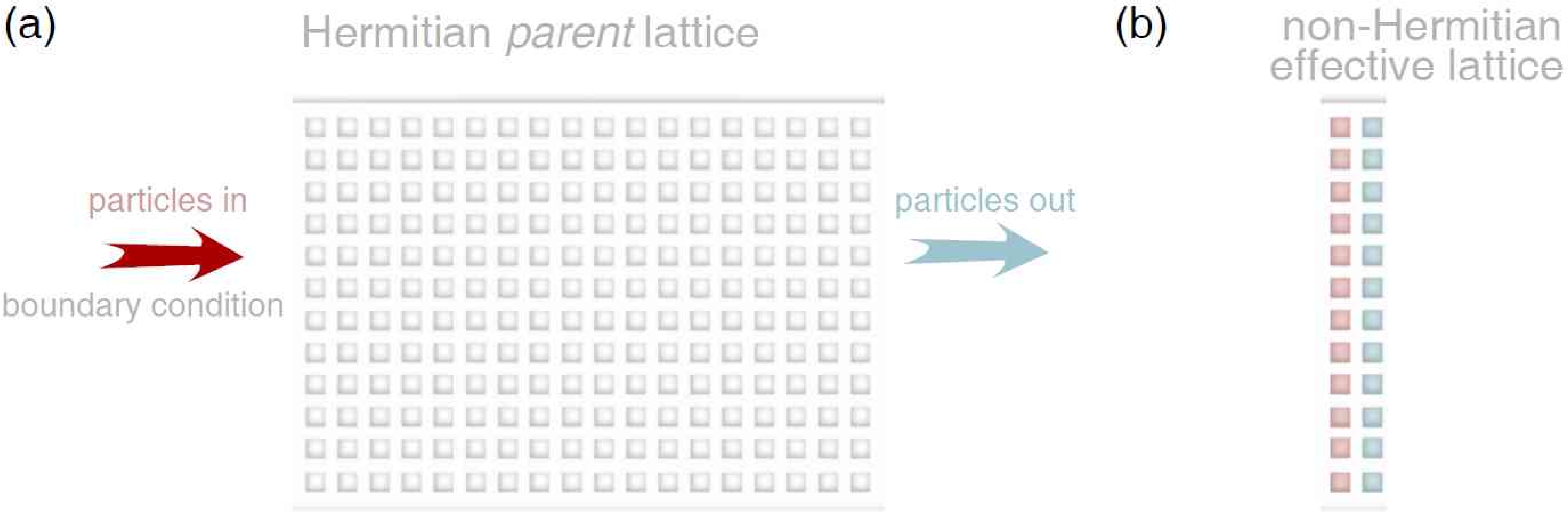
Many (unexpected) experimental confirmations!

Curiosity-driven research
with impact transcending the area.

Photo: LEFFT 2024

Phys. Rev. Lett. 129, 070401 (2022)

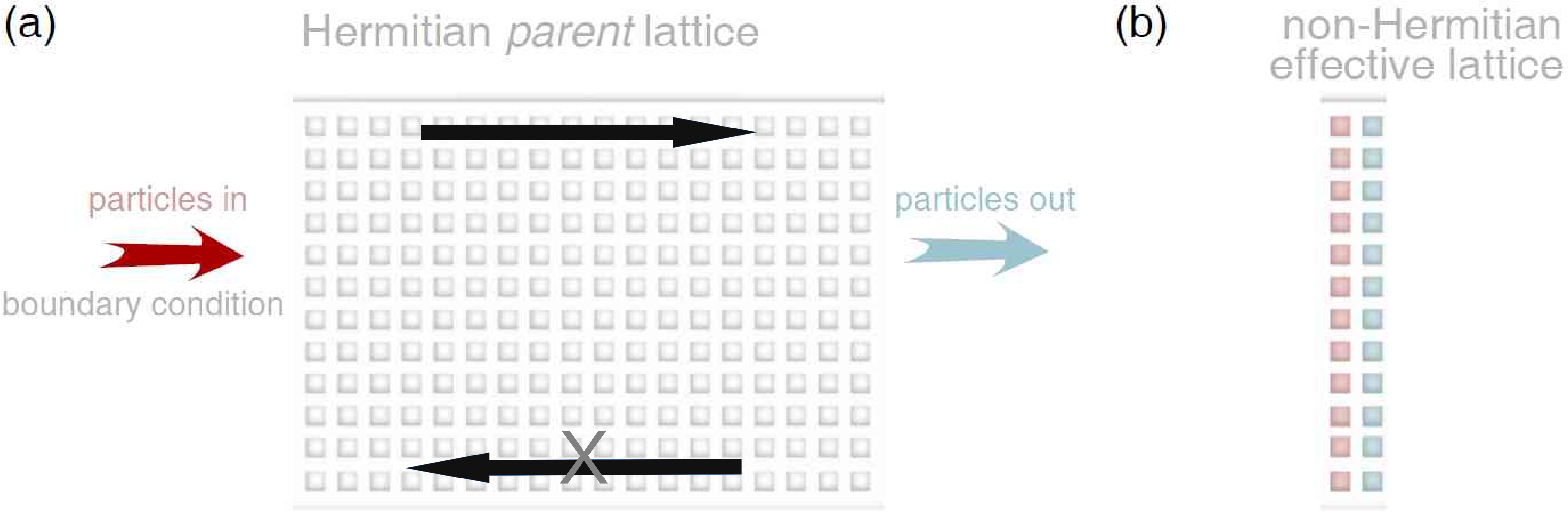
Interpreting defectiveness and connecting the dots



Non-Hermitian Hamiltonian as standard Hamiltonian *plus* boundary condition

LEFFT, J. Phys. Mater. **3**, 014002 (2020)

Interpreting defectiveness and connecting the dots



Non-Hermitian Hamiltonian as standard Hamiltonian *plus* boundary condition.
Defectiveness as lack of compatibility with the bc.

LEFFT, J. Phys. Mater. **3**, 014002 (2020)

Connections: the skin effect of reflected waves

PHYSICAL REVIEW LETTERS **129,** 086601 (2022)

Editors' Suggestion

Non-Hermitian Physics without Gain or Loss: The Skin Effect of Reflected Waves

Selma Franca,[1,*] Viktor Könye,[1] Fabian Hassler,[2] Jeroen van den Brink,[1,3] and Cosma Fulga[1]
[1]*Institute for Theoretical Solid State Physics, IFW Dresden and Würzburg-Dresden Cluster of Excellence ct.qmat, Helmholtzstrasse 20, 01069 Dresden, Germany*
[2]*JARA-Institute for Quantum Information, RWTH Aachen University, 52056 Aachen, Germany*
[3]*Institute for Theoretical Physics, TU Dresden, 01069 Dresden, Germany*

(Received 3 November 2021; accepted 27 July 2022; published 17 August 2022)

Physically, one tends to think of non-Hermitian systems in terms of gain and loss: the decay or amplification of a mode is given by the imaginary part of its energy. Here, we introduce an alternative avenue to the realm of non-Hermitian physics, which involves neither gain nor loss. Instead, complex eigenvalues emerge from the amplitudes and phase differences of waves backscattered from the boundary of insulators. We show that for any strong topological insulator in a Wigner-Dyson class, the reflected waves are characterized by a reflection matrix exhibiting the non-Hermitian skin effect. This leads to an unconventional Goos-Hänchen effect: due to non-Hermitian topology, waves undergo a lateral shift upon reflection, even at normal incidence. Going beyond systems with gain and loss vastly expands the set of experimental platforms that can access non-Hermitian physics and show signatures associated with non-Hermitian topology.

DOI: 10.1103/PhysRevLett.129.086601

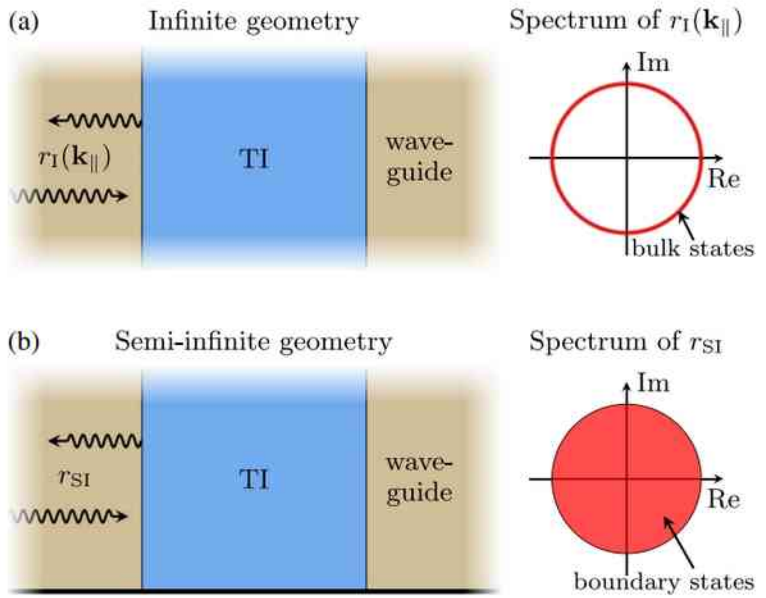


FIG. 1. Transport geometries and spectrum of the reflection matrix. Panel (a) shows the infinite geometry and panel (b) shows the semi-infinite geometry. In each panel, the spectrum of the reflection matrix is shown on the right.

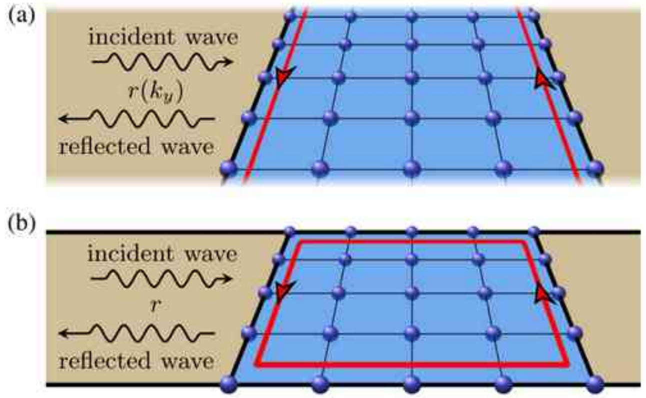
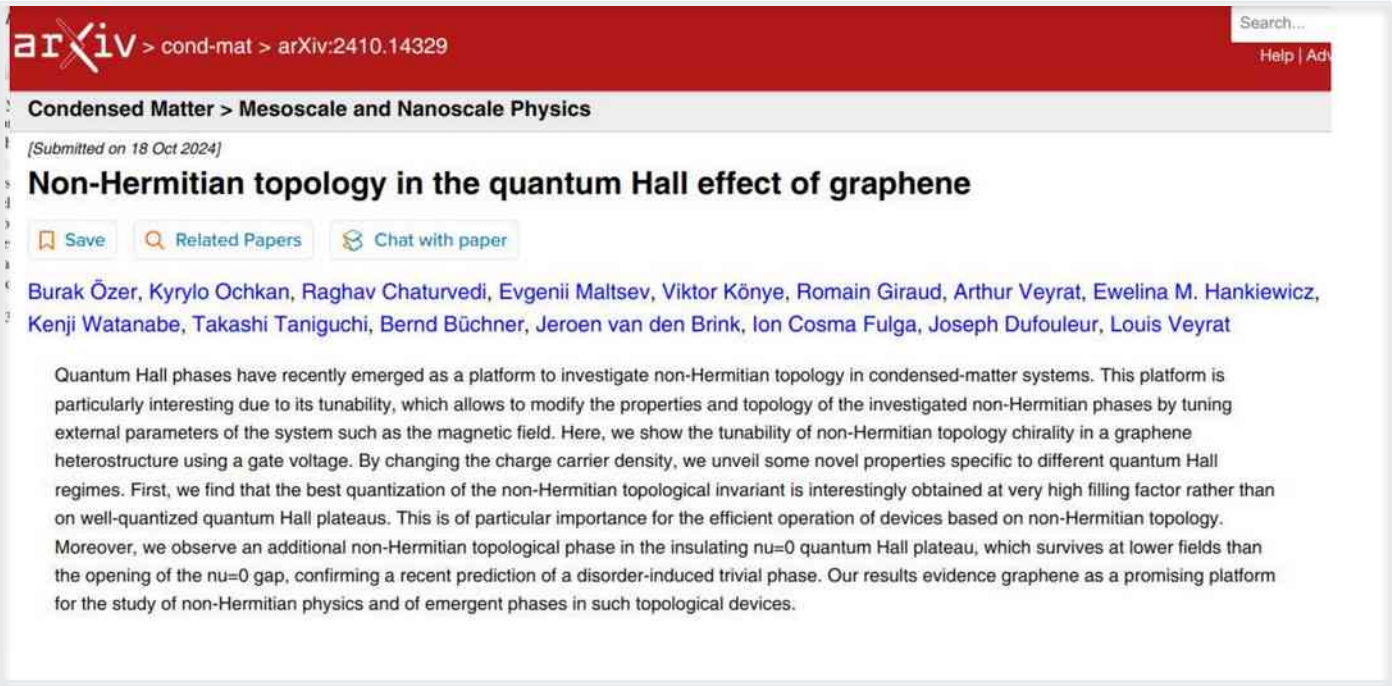


FIG. 2. Scattering setup. The Chern insulator (blue) is connected to two waveguides (beige). In panel (a), the system is infinite in the y (vertical) direction, and the reflection matrix depends on k_y. In panel (b), both the Chern insulator and the waveguides have a finite extent in the transversal direction.

And the story keeps closing loops


arXiv > cond-mat > arXiv:2410.14329

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[Submitted on 18 Oct 2024]

Non-Hermitian topology in the quantum Hall effect of graphene

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Burak Özer, Kyrylo Ochkan, Raghav Chaturvedi, Evgenii Maltsev, Viktor Könye, Romain Giraud, Arthur Veyrat, Ewelina M. Hankiewicz, Kenji Watanabe, Takashi Taniguchi, Bernd Büchner, Jeroen van den Brink, Ion Cosma Fulga, Joseph Dufouleur, Louis Veyrat

Quantum Hall phases have recently emerged as a platform to investigate non-Hermitian topology in condensed-matter systems. This platform is particularly interesting due to its tunability, which allows to modify the properties and topology of the investigated non-Hermitian phases by tuning external parameters of the system such as the magnetic field. Here, we show the tunability of non-Hermitian topology chirality in a graphene heterostructure using a gate voltage. By changing the charge carrier density, we unveil some novel properties specific to different quantum Hall regimes. First, we find that the best quantization of the non-Hermitian topological invariant is interestingly obtained at very high filling factor rather than on well-quantized quantum Hall plateaus. This is of particular importance for the efficient operation of devices based on non-Hermitian topology. Moreover, we observe an additional non-Hermitian topological phase in the insulating nu=0 quantum Hall plateau, which survives at lower fields than the opening of the nu=0 gap, confirming a recent prediction of a disorder-induced trivial phase. Our results evidence graphene as a promising platform for the study of non-Hermitian physics and of emergent phases in such topological devices.

All the material from these lectures will be here:

Most recently (2024), non-Hermitian topology has been observed directly in the quantum Hall effect of graphene — closing the loop back to where this mini-course began.

From SSH, to Haldane, to Kane–Mele, to driven & non-Hermitian phases — and back to the quantum Hall effect.

B. Özer et al., arXiv:2410.14329 (2024)