

Image: Walter Donega



Quantum Hall effects

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QUANTUM MATTER HORIZONS SCHOOL · 2026



UNIVERSIDAD
DE CHILE

An aerial photograph of a winding asphalt road that snakes through a dense forest. The road has white dashed lines and a few small cars are visible. The surrounding trees are mostly green, with some autumn-colored trees in shades of orange and yellow visible in the lower-left corner.

Outline of this talk

Back in 1879: The Hall effect

The **integer** Quantum Hall effect

Topological connections

The **anomalous** Quantum Hall effect

The Quantum **Spin** Hall effect

The **3D** Quantum Hall effect

Light-induced Quantum Hall effect

Final remarks

Outline of this talk

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Final remarks



Outline of this talk

Quantum Hall effects



Image: Calin Stan

artificial systems
under
extreme conditions

materials
and
applications

Macroscopic quantum phenomena

An aerial photograph of a winding asphalt road that snakes through a dense forest. The road has white dashed lines and a few small cars are visible. The surrounding trees are mostly green, with some patches of autumn-colored foliage in shades of orange and red. The road's path is highly irregular, forming a series of loops and turns that mimic the shape of a stylized letter 'S' or a complex knot.

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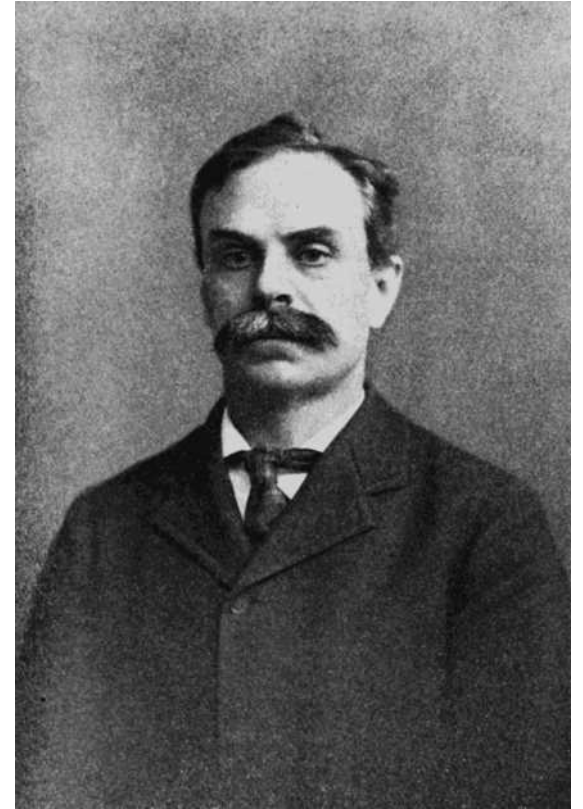
Light-induced Quantum Hall effect

Final remarks

Back to 1879

As a doctoral student at John Hopkins, **Edwin Hall** (24) was puzzled by a statement in James Clerk Maxwell's two-volume "A Treatise on Electricity and Magnetism" from 1873 and was set to prove him wrong.

The result is known today as **the Hall effect**.



Edwin H. Hall

Back to 1879

The Hall effect was discovered by Hall in **1879**, while working on his doctoral thesis in Physics at John Hopkins.

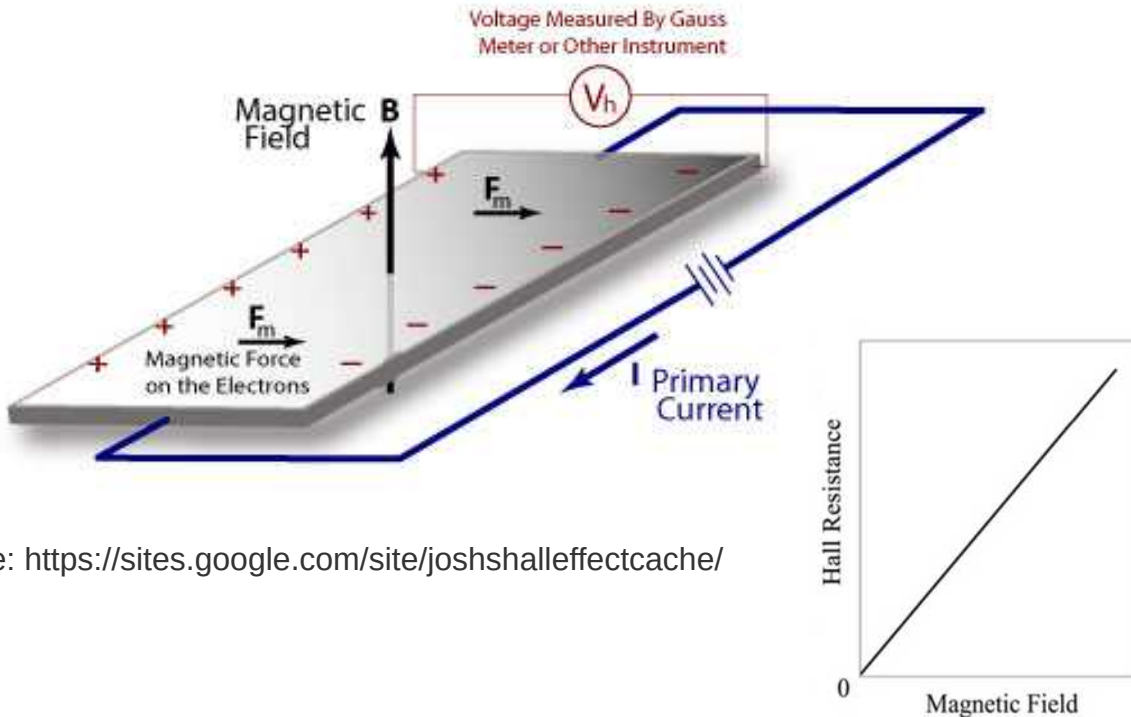
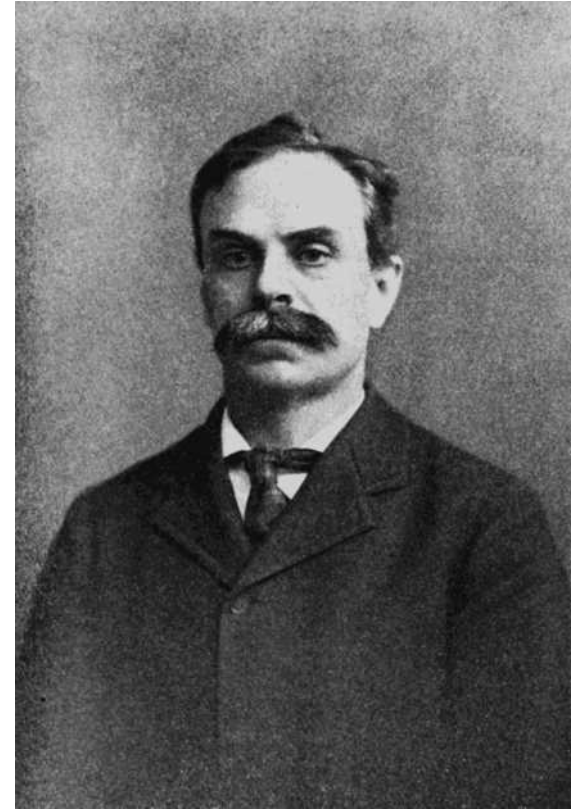


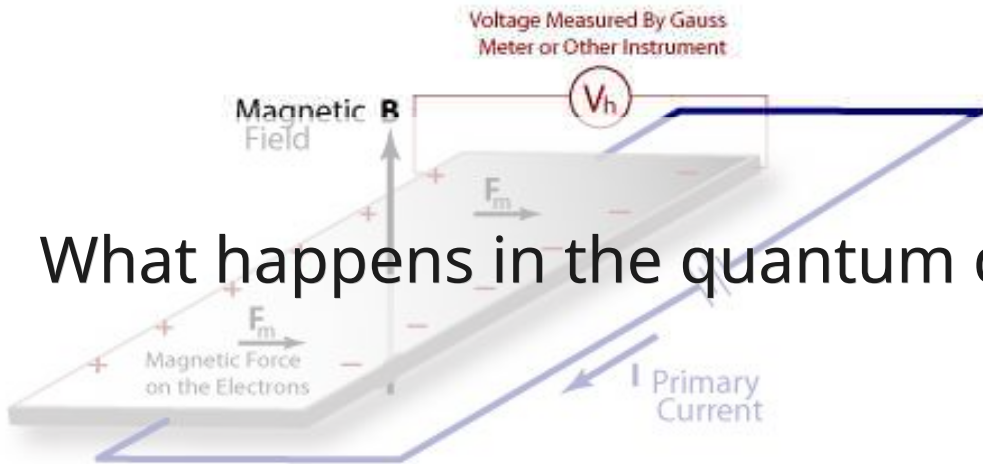
Image: <https://sites.google.com/site/joshshalleffectcache/>



Edwin H. Hall

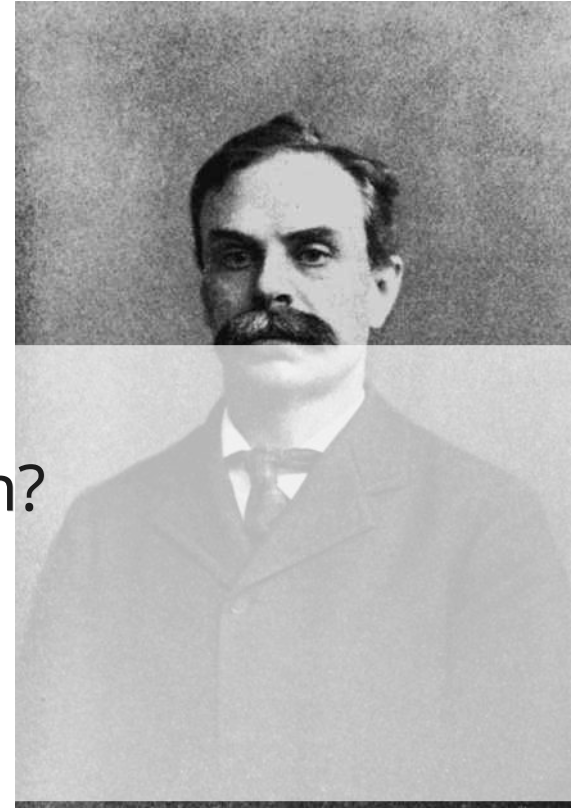
Back to 1879

The Hall effect was discovered by Hall in **1879**, while working on his doctoral thesis in Physics at John Hopkins.



What happens in the quantum domain?

Image: <https://sites.google.com/site/joshshalleffectcache/>



Edwin H. Hall

An aerial photograph of a dark, winding road that snakes through a dense forest covered in a thick layer of snow. The road has several sharp turns and loops, creating a complex, organic shape. The surrounding trees are mostly bare, with their branches dusted in white. The overall scene is serene and wintry.

Outline of this talk

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The 3D Quantum Hall effect

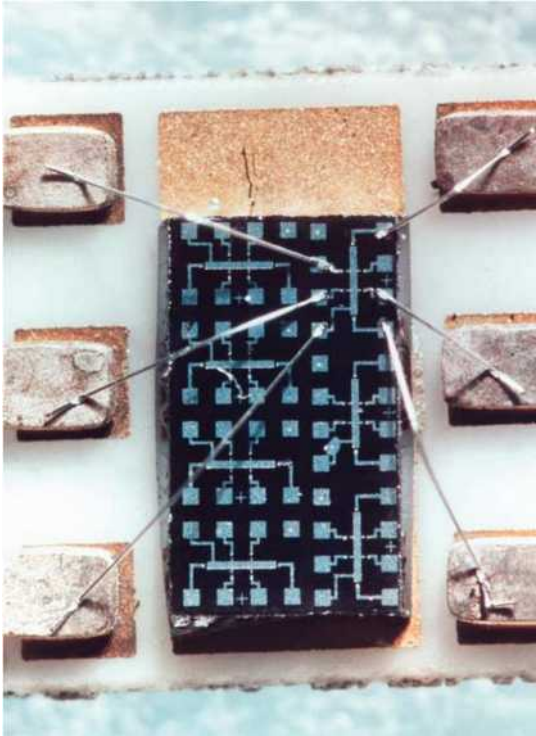
Light-induced Quantum Hall effect

Final remarks



101 years later... (Grenoble 1980)

An unlikely finding



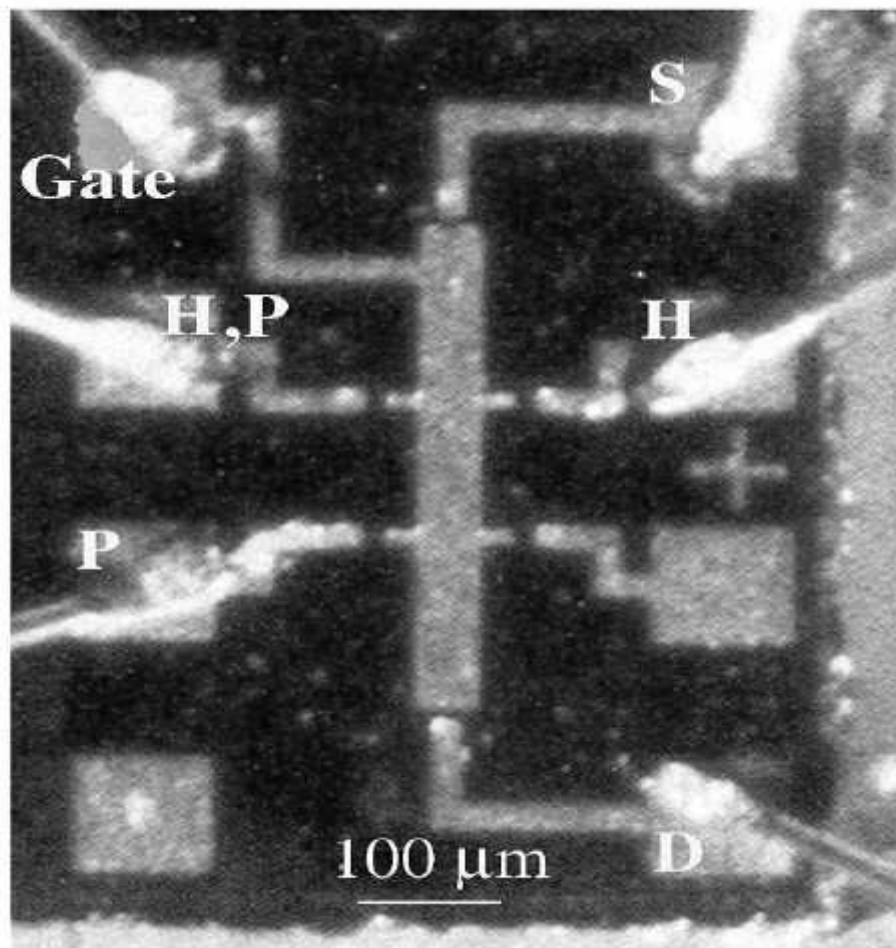
Silicon Field Effect Transistor with additional probes for measuring the Hall and longitudinal conductances.

2D electron gas at the extreme quantum limit



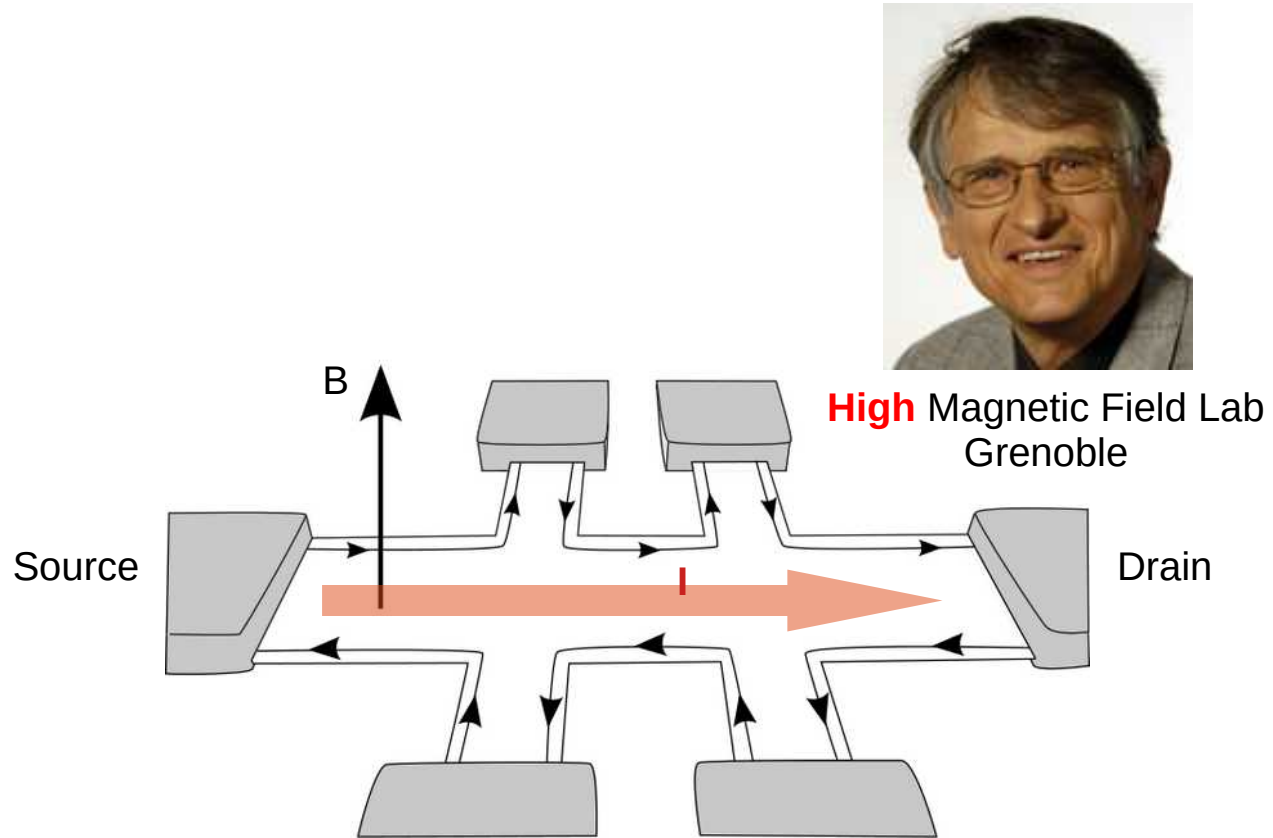
High Magnetic Field Lab, Grenoble

K. von Klitzing, Phil. Trans. R. Soc. A **363**, 2203 (2005).



Multiterminal setup

Fix the current I ,
measure different voltages
among the terminals

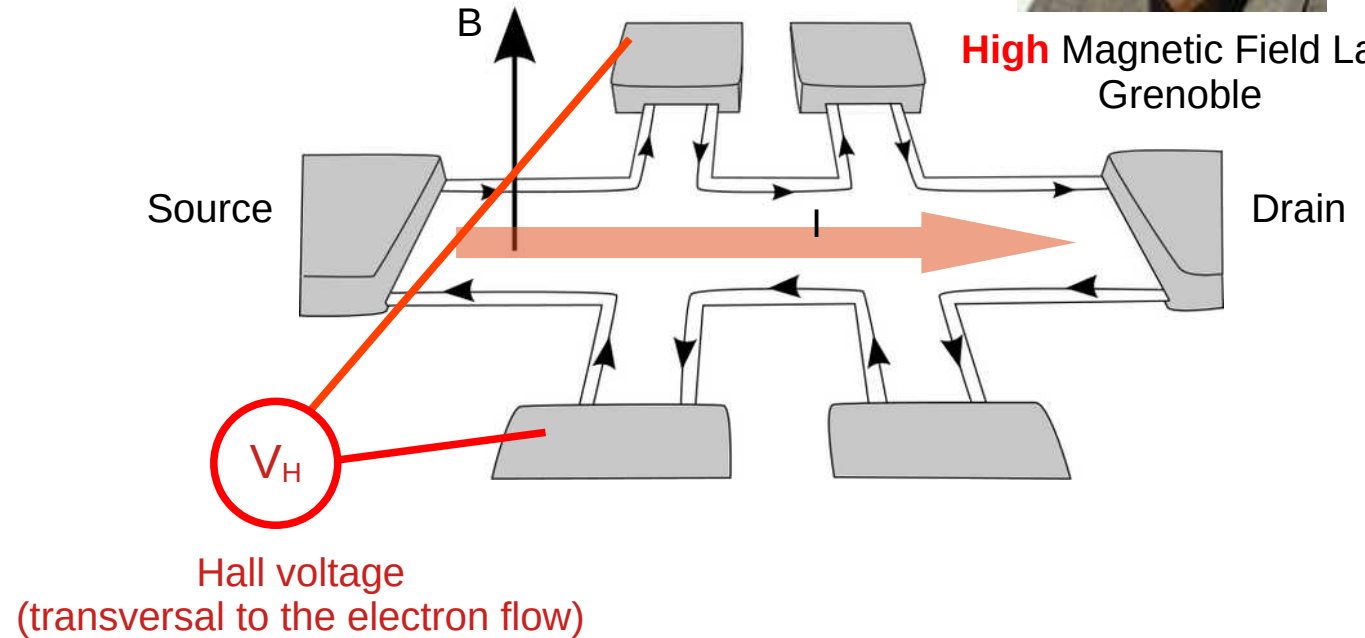




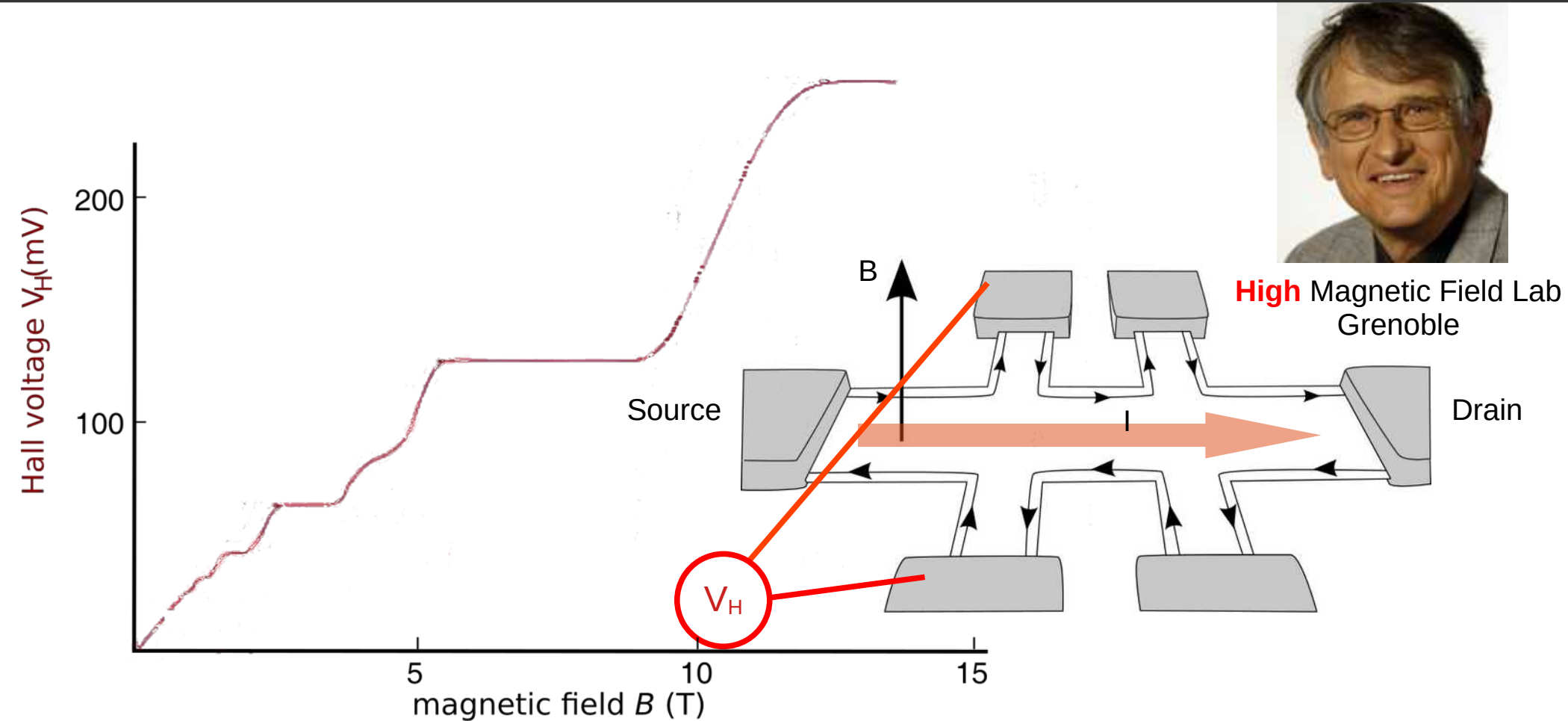
High Magnetic Field Lab
Grenoble

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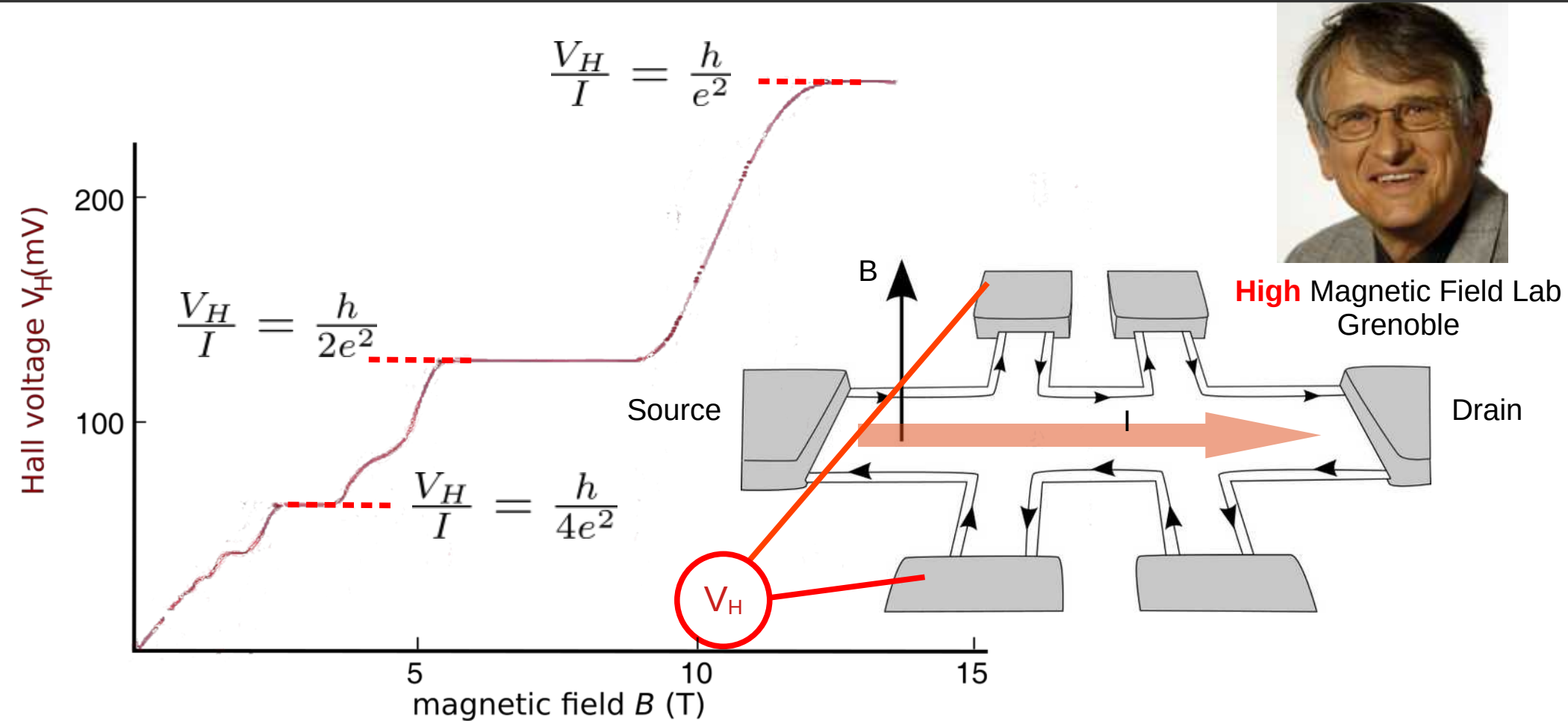


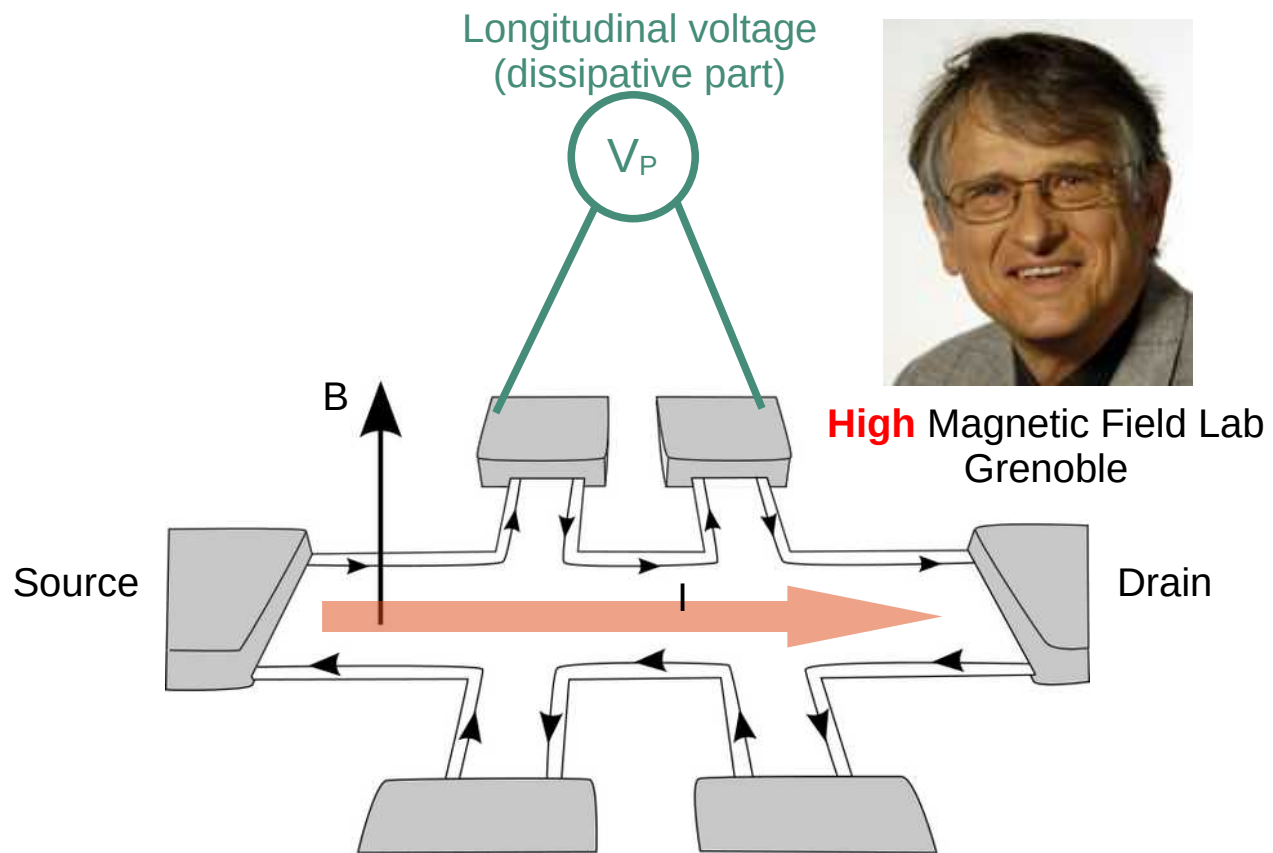
The integer Quantum Hall Effect

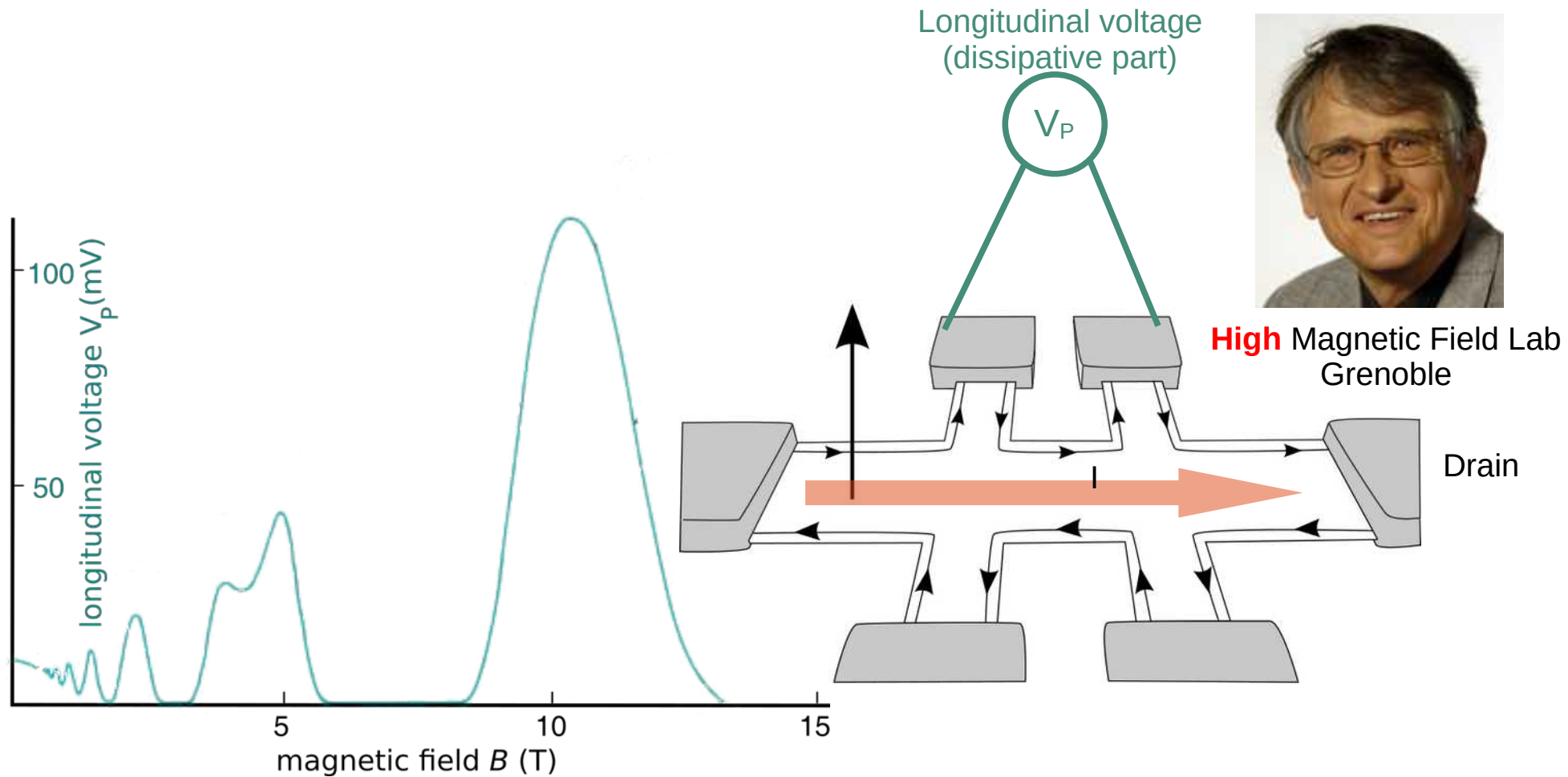


K. von Klitzing, Phil. Trans. R. Soc. A **363**, 2203 (2005).

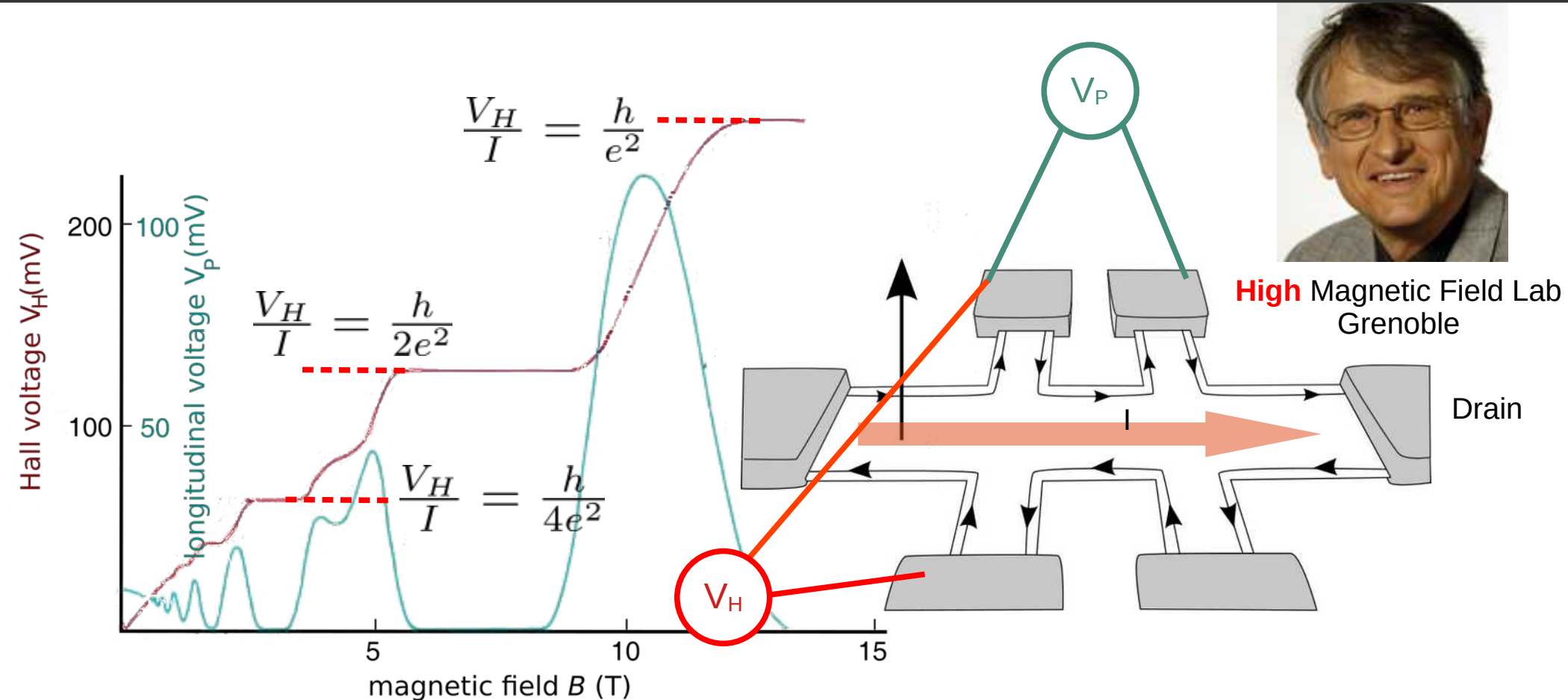
The integer Quantum Hall Effect







The integer Quantum Hall Effect



The integer Quantum Hall Effect

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing, G. Dorda, and M. Pepper
Phys. Rev. Lett. **45**, 494 – Published 11 August 1980

Physics See Focus story: *Landmarks—Accidental Discovery Leads to Calibration Standard*

An article within the collection: *Letters from the Past – A PRL Retrospective*



Notes 4/5.2.1980

Diagram of a rotating sample holder with pin connections. Labels include: $q_H(C_1)$, $q_H(D)$, $q_H(S)$, $4C(S)$, $4C(D)$, $4C(S)$, $4C(D)$, E , S , G , D , q , $q_{S,D}=6$, $pin\ connections$.

Equations:

$$E_H = R_H \cdot D \cdot j = \frac{1}{n \cdot e} \cdot B \cdot \frac{I}{b}$$

$$U_H = \frac{B}{n \cdot e} \cdot I$$

$$U_H = \frac{2 \pi \hbar^2 \cdot B \cdot I}{e \cdot e \cdot B} = \left(\frac{h}{e^2} \cdot I \right)$$

Diagram of a rectangular sample with dimensions $25.76 \mu m$ and 25813 .

Equation:

$$\frac{\phi}{4\pi} \frac{t_c}{e^2} = \rho_{xy} = \frac{\alpha}{2} \cdot \sqrt{\frac{\mu_0}{\epsilon_0}} \Rightarrow 25813 \Omega$$

notes of the phone call to PTB
PTB 531/5929 (5.2.1980)

von Klitzing constant $R_K = h/e^2 = 25812.807557(18) \Omega$

Equations:

$$\mu_0 = 4\pi \cdot 10^{-9} \frac{Vs}{Am}$$

$$\epsilon_0 = 0.8854 \cdot 10^{-12} \frac{As}{Vm}$$

$$\sqrt{\frac{\epsilon_0}{\mu_0}} = 2.65 \cdot 10^{-3} \Omega^{-1}$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7 \Omega$$

Quantized resistances with and without the input resistance of the x-y recorder:

Quantized resistances	with and without the input resistance of the x-y recorder
25813 Ω : N	25813 $\rightarrow$ 25763.46
1 M Ω parallel	12906.5
	6453.25
	3226.63
	326.25
	2151.08
	2146.47

The integer Quantum Hall Effect

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

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Notes 4/5.2.1980

rotating sample holder

pin connections

$$E_H = R_H \cdot D \cdot j = \frac{1}{n \cdot e} \cdot B \cdot \frac{I}{b}$$

$$U_H = \frac{B}{n \cdot e} \cdot I$$

$$U_H = \frac{2 \pi \hbar}{e \cdot e \cdot B} \cdot I = \left(\frac{h}{e^2} \cdot I \right)$$

$N = \frac{eB}{2\pi k} \quad (g_s \cdot g_v = 1)$

Josephson

$$\frac{\phi_0}{2\pi} = \frac{h c}{2 e} = \frac{2.067833848 \times 10^{-15} \text{ Vs}}{2} \Rightarrow 25813 \Omega$$

notes of the phone call to PTB

von Klitzing constant $R_K = h/e^2 = 25812.807557(18) \Omega$

$\epsilon_0 = 0.8854 \cdot 10^{-12} \frac{Fs}{Vm}$

$\sqrt{\frac{\epsilon_0}{\mu_0}} = 2.99792458 \cdot 10^8 \text{ m/s}$

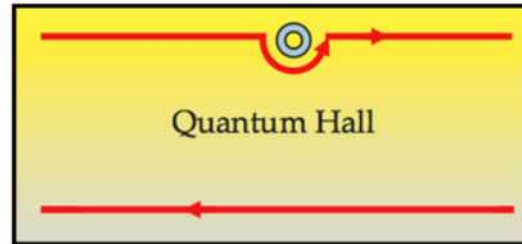
$\sqrt{\frac{\epsilon_0}{\mu_0}} = 376.73 \Omega$

quantized resistances with and without the input resistance of the x-y recorder

25813 Ω : N	1 M Ω parallel	25813 $\rightarrow$ 2562.46	12906.5	12742.04
6453.25	6411.87	3226.63	3246.25	2146.47
2151.08				

How to explain these results?

- ➔ Nearly exact quantization of the Hall conductance (universal constants) with vanishing longitudinal conductance.
- ➔ Robustness to disorder and imperfections (the samples were not controlled for defects).



Satija, "Butterfly in the Quantum World" (2016)

The integer Quantum Hall Effect

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

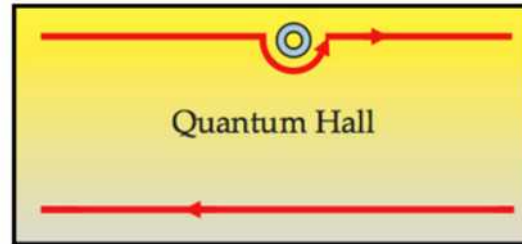
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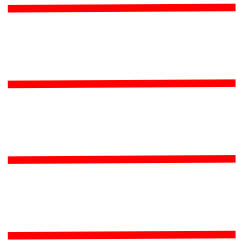
Satija, "Butterfly in the Quantum World" (2016)

A view from the edge

A topological view

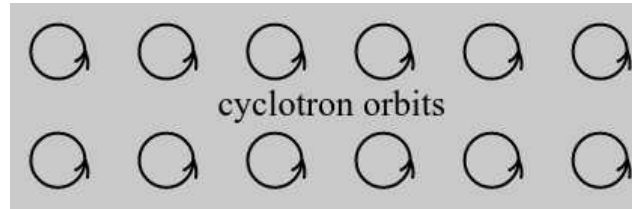
The integer Quantum Hall effect: A view from the edge

E ↑



Landau levels for the “bulk”

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega_c$$

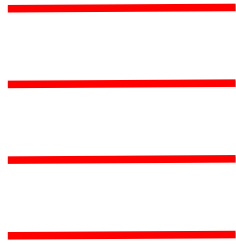


Interpretation of flat
(massively degenerate) bands.

Satija, “Butterfly in the Quantum World” (2016)

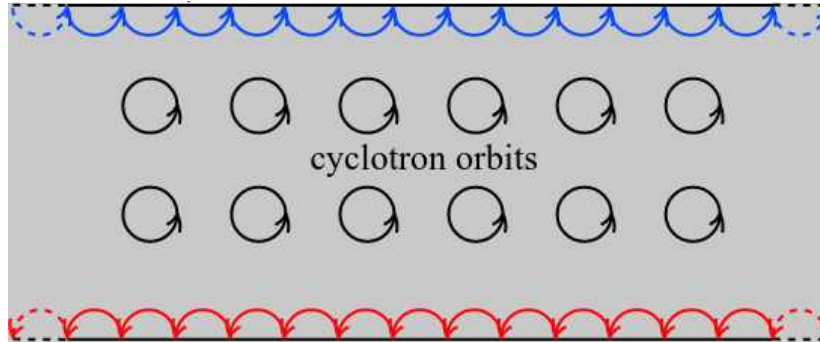
The integer Quantum Hall effect: A view from the edge

E ↑



Landau levels for the “bulk”

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$



But in a **finite system**...

Satija, “Butterfly in the Quantum World” (2016)

The integer Quantum Hall effect: A view from the edge

Landau levels in real space

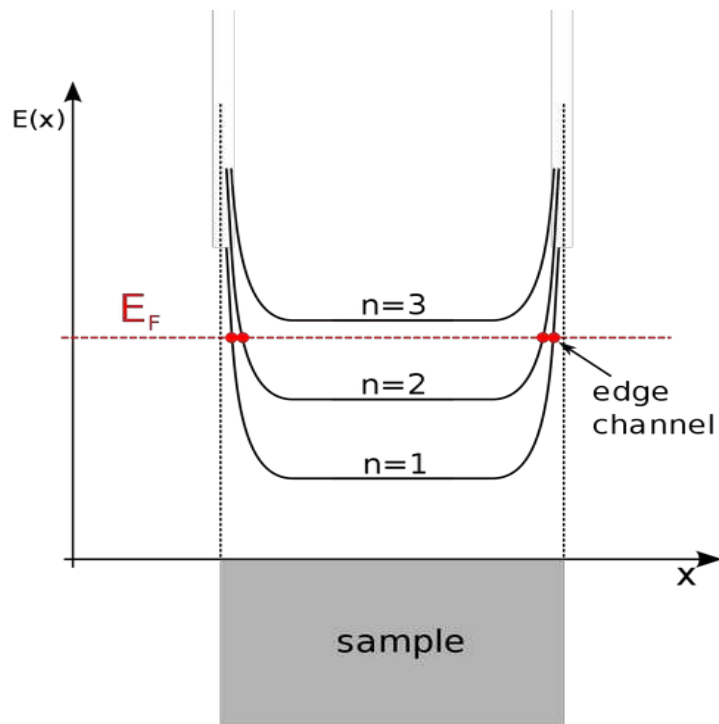
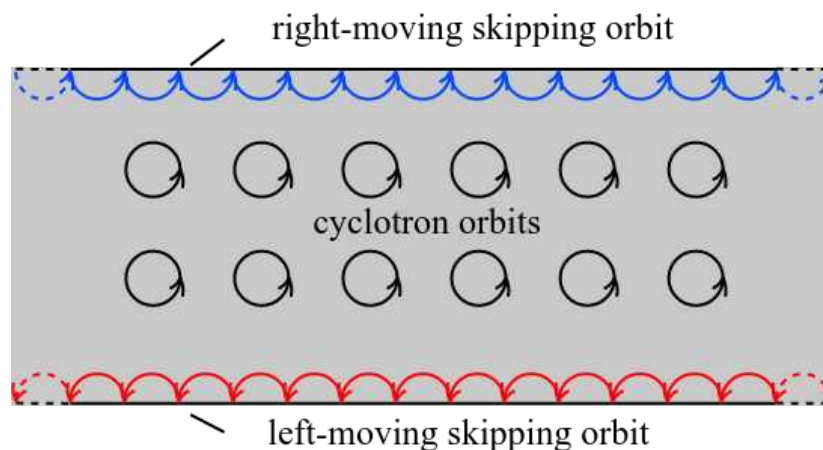


Image: wikipedia

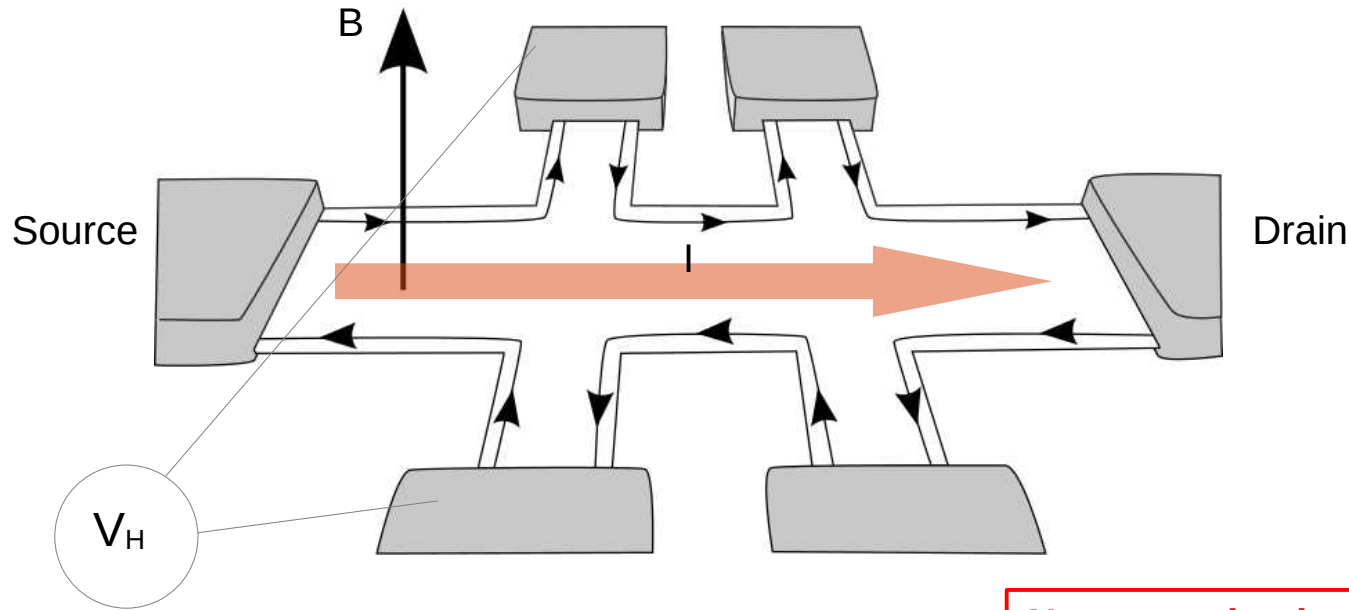
Landau levels for the “bulk”

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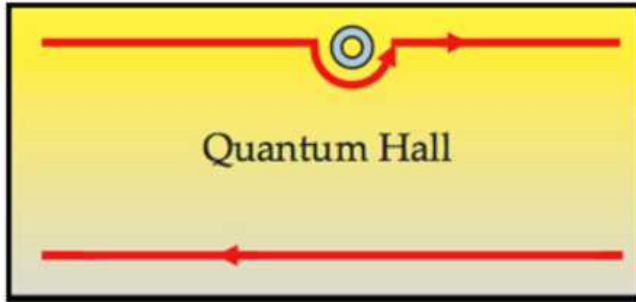
Satija, "Butterfly in the Quantum World" (2016)

The integer Quantum Hall effect: A view from the edge



No way to backscatter!!!
> Robust conductance.
(no matter how rough the edges)

The integer Quantum Hall effect: A view from the edge



Satija, "Butterfly in the Quantum World" (2016)



Mind the gap

No way to backscatter!!!
> Robust conductance.
(no matter how rough the edges)

The integer Quantum Hall effect

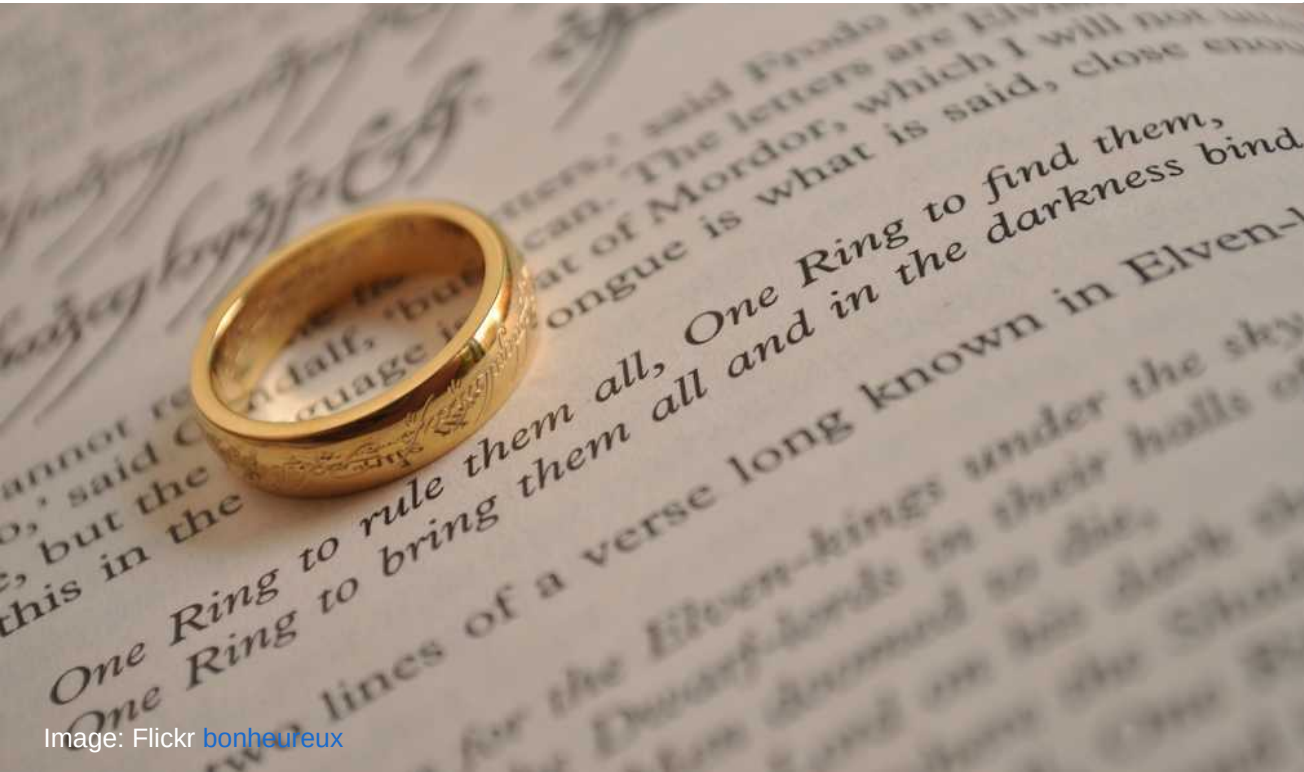


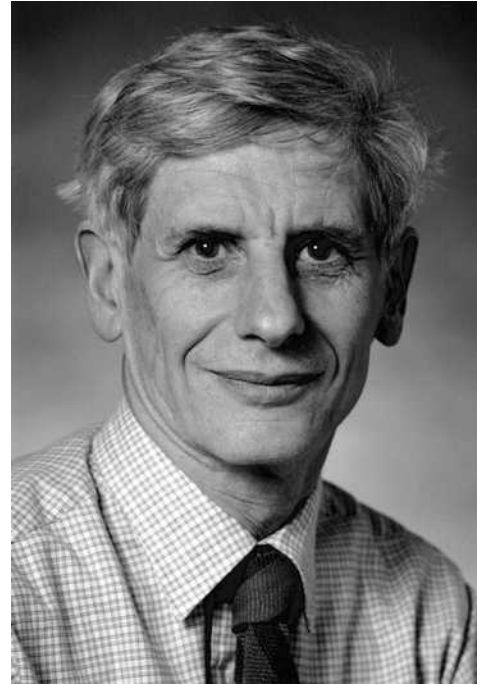
Image: Flickr [bonheureux](#)

A topological argument
to connect the edge
with the bulk

The integer Quantum Hall effect: A topological argument to tie the edge with the bulk

In a famous paper that appeared in 1982, Thouless, Kohmoto, Nightingale and den Nijs (usually referred as the TKNN paper), **they showed that the Hall conductivity is precisely the Chern number**. To such end they used linear response theory showing that the Kubo formula for the Hall conductivity gives precisely the Chern number.

Phys. Rev. Lett. **49**, 405 (1982)



David J. Thouless
Source: Wikipedia

The integer Quantum Hall effect: A topological argument to tie the edge with the bulk

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,^(a) M. P. Nightingale, and M. den Nijs

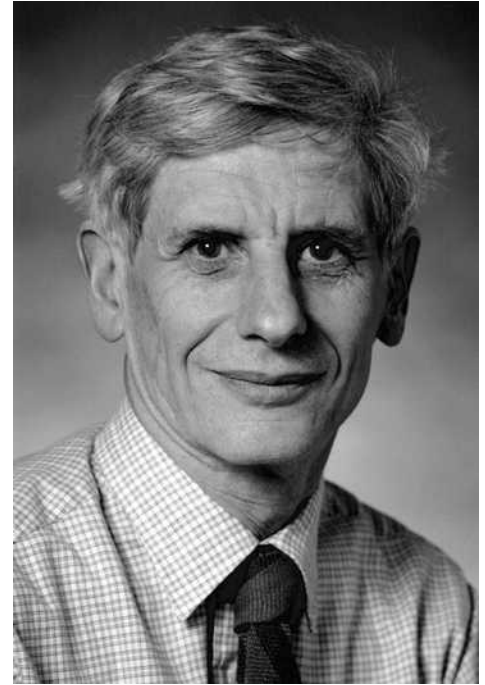
Department of Physics, University of Washington, Seattle, Washington 98195

(Received 30 April 1982)

The Hall conductance of a two-dimensional electron gas has been studied in a uniform magnetic field and a periodic substrate potential U . The Kubo formula is written in a form that makes apparent the quantization when the Fermi energy lies in a gap. Explicit expressions have been obtained for the Hall conductance for both large and small $U/\hbar\omega_c$.

PACS numbers: 72.15.Gd, 72.20.Mg, 73.90.+b

Phys. Rev. Lett. **49**, 405 (1982)

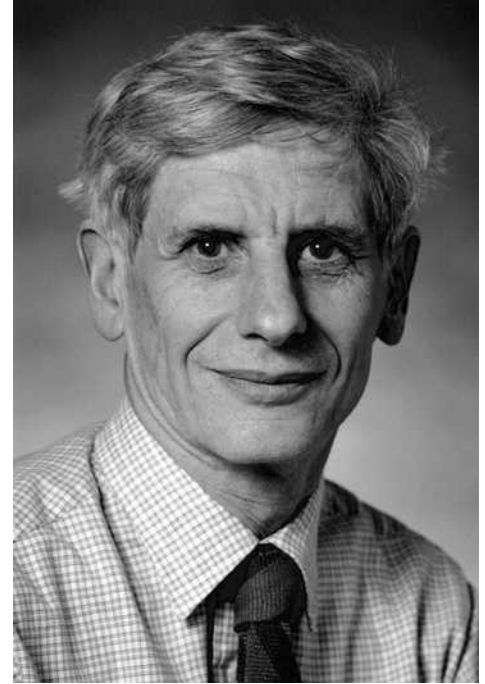


David J. Thouless
Source: Wikipedia

The integer Quantum Hall effect: A topological argument to tie the edge with the bulk

The Hall conductivity is given by the Chern number, a *“topological invariant”*, that can take only integer values.

information complementary to the bandstructure



David J. Thouless
Source: Wikipedia

The integer Quantum Hall effect: A topological argument to tie the edge with the bulk

The bulk is characterized by an integral property, a “topological invariant”.

(non-local)

$$\int_{\text{BZ}} F$$

Counts the number of “knots” in the bulk wave-functions.

The integer Quantum Hall effect: A topological argument to tie the edge with the bulk

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Moore, Nature **464**, 164 (2010).



Not all insulators are equal.

The integer Quantum Hall effect: A topological argument to tie the edge with the bulk

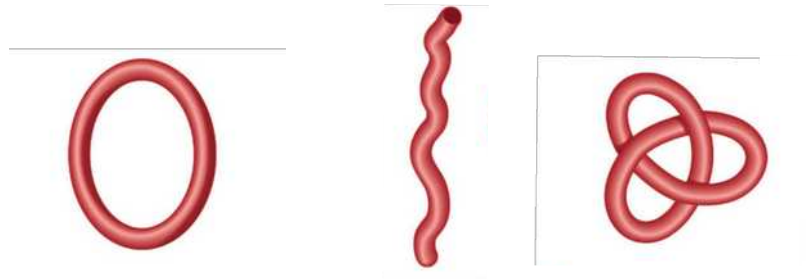
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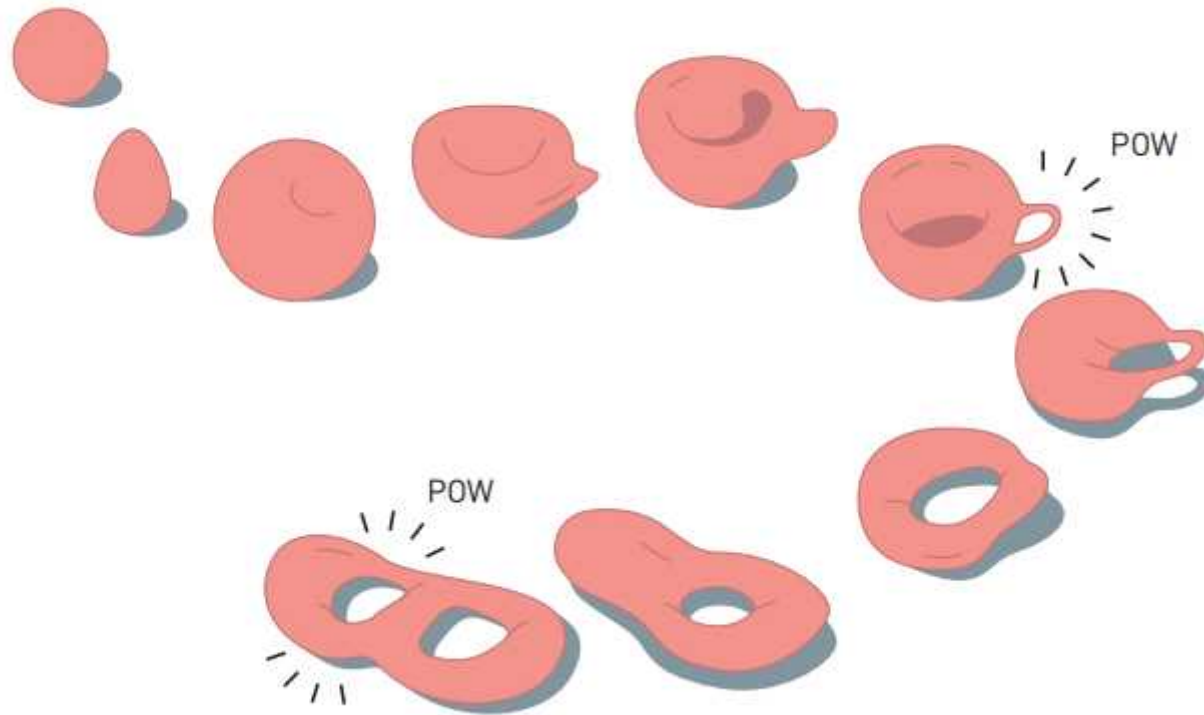
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The integer Quantum Hall effect: A topological argument to tie the edge with the bulk



The Anomalous Quantum Hall effect

VOLUME 61, NUMBER 18

PHYSICAL REVIEW LETTERS

31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the “Parity Anomaly”

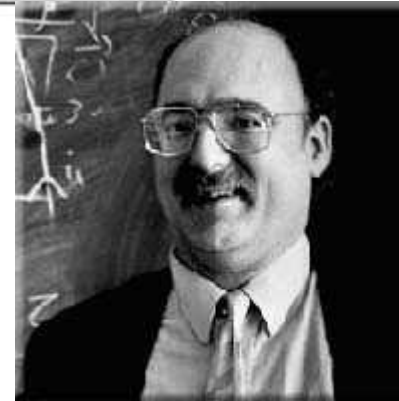
F. D. M. Haldane

Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called “parity anomaly” of (2+1)-dimensional field theories.

PACS numbers: 05.30.Fk, 11.30.Rd



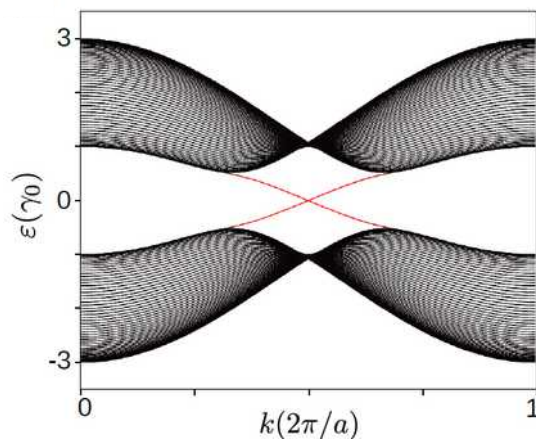
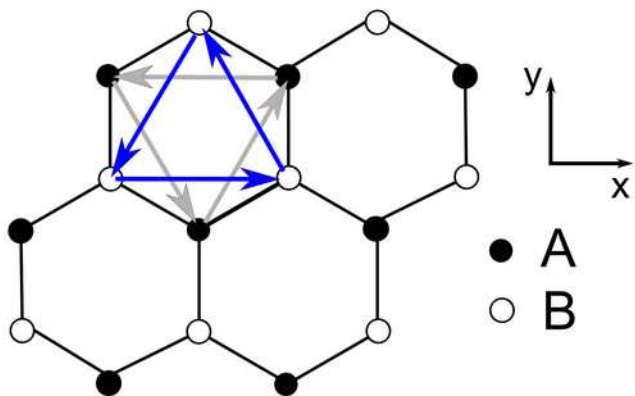
The Anomalous Quantum Hall effect

$$\mathcal{H} = \Delta \sum_j \xi_j c_j^\dagger c_j + \gamma_0 \sum_{\langle i,j \rangle} c_i^\dagger c_j + \gamma_2 \sum_{\langle\langle i,j \rangle\rangle} e^{-i\epsilon_{ij}\phi} c_i^\dagger c_j$$

$\xi_j = +1$ when $j \in A$, $\xi_j = -1$ when $j \in B$

“mass” term breaking
sublattice/inversion
symmetry

Haldane term
second nearest neighbors
(complex !)



The Quantum Spin-Hall effect

PRL **95**, 226801 (2005)

PHYSICAL REVIEW LETTERS

week ending
25 NOVEMBER 2005

Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele

Dept. of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 29 November 2004; published 23 November 2005)

We study the effects of spin orbit interactions on the low energy electronic structure of a single plane of graphene. We find that in an experimentally accessible low temperature regime the symmetry allowed spin orbit potential converts graphene from an ideal two-dimensional semimetallic state to a quantum spin Hall insulator. This novel electronic state of matter is gapped in the bulk and supports the transport of spin and charge in gapless edge states that propagate at the sample boundaries. The edge states are nonchiral, but they are insensitive to disorder because their directionality is correlated with spin. The spin and charge conductances in these edge states are calculated and the effects of temperature, chemical potential, Rashba coupling, disorder, and symmetry breaking fields are discussed.

DOI: [10.1103/PhysRevLett.95.226801](https://doi.org/10.1103/PhysRevLett.95.226801)

PACS numbers: 73.43.-f, 72.25.Hg, 73.61.Wp, 85.75.-d

The Quantum Spin-Hall effect

BUT the **spin-orbit interaction** allows for a different kind of **gap-opening term** :

Spin-orbit
coupling

$$-i\gamma_{SO} \sum_{\langle\langle i,j \rangle\rangle, s_z} \nu_{i,j} s_z |i, s_z\rangle \langle j, s_z|$$

± 1
depending
on the spin

. Preserves S_z

. Preserves TRS
(this is, none of the
symmetries in graphene
are broken)

Two copies of the
Haldane Hamiltonian, one
for each spin with a
reversed sign !

The Quantum Spin-Hall effect

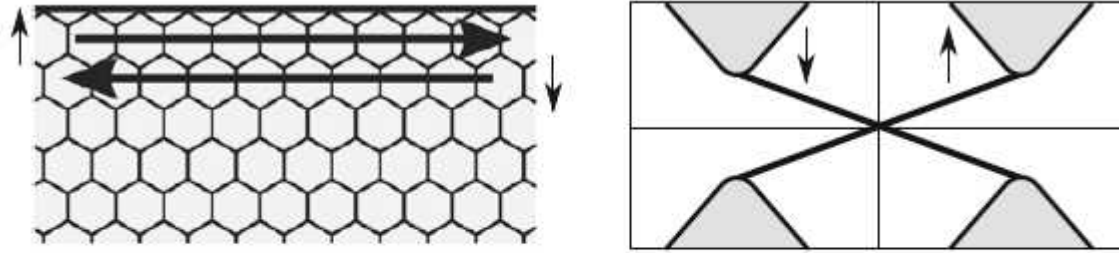


Fig. from “Top. Insulators” by Franz and Molenkamp (Eds) 2013

Connection with the Berry phase

Proc. R. Soc. Lond. A **392**, 45–57 (1984)

Printed in Great Britain

Quantal phase factors accompanying adiabatic changes

BY M. V. BERRY, F.R.S.

*H. H. Wills Physics Laboratory, University of Bristol,
Tyndall Avenue, Bristol BS8 1TL, U.K.*

(Received 13 June 1983)



Connection with the Berry phase

$$\mathcal{H}(\boldsymbol{\lambda})$$

$$\boldsymbol{\lambda}(t) = (\lambda_1(t), \lambda_2(t), \dots, \lambda_n(t))$$

Connection with the Berry phase

$$\mathcal{H}(\boldsymbol{\lambda}) \qquad \boldsymbol{\lambda}(t) = (\lambda_1(t), \lambda_2(t), \dots, \lambda_n(t))$$

$$|\psi_0(\boldsymbol{\lambda})\rangle \xrightarrow{t} e^{i\varphi(t)} |\psi_0(\boldsymbol{\lambda})\rangle$$

The integer Quantum Hall effect: Connection with the Berry phase

$$\mathcal{H}(\boldsymbol{\lambda})$$

$$\boldsymbol{\lambda}(t) = (\lambda_1(t), \lambda_2(t), \dots, \lambda_n(t))$$

$$|\psi_0(\boldsymbol{\lambda})\rangle \xrightarrow{t} e^{i\varphi(t)} |\psi_0(\boldsymbol{\lambda})\rangle$$

$$\varphi(t) - \varphi(0) = \int_{\boldsymbol{\lambda}(0)}^{\boldsymbol{\lambda}(t)} \langle \psi_0(\boldsymbol{\lambda}) | i \nabla_{\boldsymbol{\lambda}} | \psi_0(\boldsymbol{\lambda}) \rangle \cdot d\boldsymbol{\lambda} - \frac{1}{\hbar} \int_0^t dt' E_0(t')$$

geometrical phase

Dynamical phase

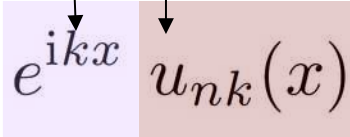
Connection with the Berry phase

In crystals one can define the same concept with the wavevector $\mathbf{k}$ taking the role of the parameter space, and the Bloch wave-functions taking the role of the states.

Wave function

$$\psi_{n\mathbf{k}}(x) = e^{i\mathbf{k}x} u_{n\mathbf{k}}(x)$$

Quasi momentum Bloch wave

The equation $\psi_{n\mathbf{k}}(x) = e^{i\mathbf{k}x} u_{n\mathbf{k}}(x)$ is shown. The term $e^{i\mathbf{k}x}$ is highlighted with a light purple background, and the term $u_{n\mathbf{k}}(x)$ is highlighted with a light red background. An arrow labeled "Quasi momentum" points to the $e^{i\mathbf{k}x}$ term, and an arrow labeled "Bloch wave" points to the $u_{n\mathbf{k}}(x)$ term.

$$\mathcal{H}_{\mathbf{k}} |u_{n\mathbf{k}}\rangle = \varepsilon_{n\mathbf{k}} |u_{n\mathbf{k}}\rangle$$

J. Zak, "Berry's phase for energy bands in solids", Phys. Rev. Lett. **62**, 2747 (1989)

Connection with the Berry phase

A few consequences:

- The Chern number is just the Berry phase for the corresponding Bloch band divided by 2π .

Connection with the Berry phase

A few consequences:

- The Chern number is just the Berry phase for the corresponding Bloch band divided by 2π .
- A related quantity: the Berry curvature, acts much like a magnetic field but in $\mathbf{k}$ -space.
- Anomalous velocity and **valley-Hall effect**.

transverse
motion

$$\dot{\mathbf{r}} = \frac{1}{\hbar} \frac{\partial \varepsilon_n(\mathbf{k})}{\partial \mathbf{k}} - \dot{\mathbf{k}} \times \boldsymbol{\Omega}_n(\mathbf{k}),$$
$$\hbar \dot{\mathbf{k}} = -e\mathbf{E}(\mathbf{r}) - e\dot{\mathbf{r}} \times \mathbf{B}(\mathbf{r}),$$

Quantum valley-Hall effect

PRL **99**, 236809 (2007)

PHYSICAL REVIEW LETTERS

week ending
7 DECEMBER 2007



Valley-Contrasting Physics in Graphene: Magnetic Moment and Topological Transport

Di Xiao,* Wang Yao,* and Qian Niu

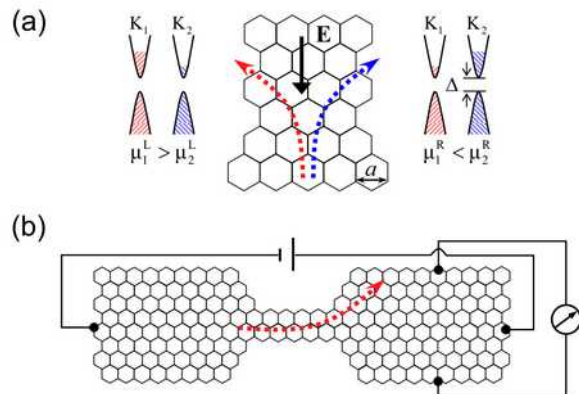
Department of Physics, The University of Texas, Austin, Texas 78712-0264, USA

(Received 11 September 2007; published 7 December 2007)

We investigate physical properties that can be used to distinguish the valley degree of freedom in systems where inversion symmetry is broken, using graphene systems as examples. We show that the pseudospin associated with the valley index of carriers has an intrinsic magnetic moment, in close analogy with the Bohr magneton for the electron spin. There is also a valley dependent Berry phase effect that can result in a valley contrasting Hall transport, with carriers in different valleys turning into opposite directions transverse to an in-plane electric field. These effects can be used to generate and detect valley polarization by magnetic and electric means, forming the basis for the valley-based electronics applications.

DOI: 10.1103/PhysRevLett.99.236809

PACS numbers: 73.63.-b, 75.75.+a, 85.35.-p



A valley of opportunities

After nearly two decades of graphene research, condensed-matter physicists **Luis Foa Torres** and **Sergio O Valenzuela** delve into the ongoing mystery of the material's perplexing non-local response and the "valley Hall effect"

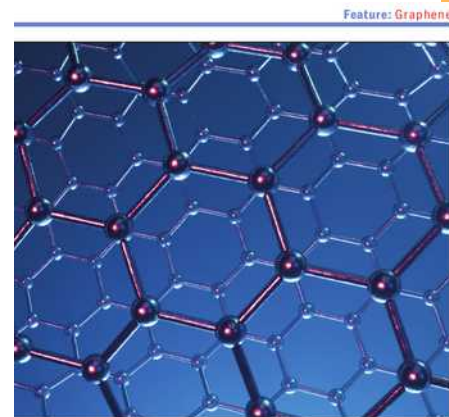
Since it was first successfully isolated in 2004, the "wonder material" graphene, with its honeycomb-like 2D structure and its wide gamut of interesting properties, has been keenly studied by material scientists. In recent years, however, the focus has shifted from simply exploring its physical properties to using graphene as a platform for testing and realizing a plethora of novel ideas. This was especially apparent with the "From graphene physics to graphene for physics" theme of the 2017 Solway workshop on graphene. And, like a relentless spring of fresh water from different sources, graphene is still unveiling surprises.

Today, there are numerous areas of research in the field where lively discussions are being had. Superconductivity in rotated graphene layers, for example, is of interest to many researchers, as it may open the door to a better understanding of superconductivity as a whole. But a key puzzle in graphene is the material's perplexing non-local response, which has seeded a series of studies addressing the use of gra-

phene's "valley degree of freedom". Electrons have fundamental properties – such as "charge" and "spin" – which allow for a number of emergent phenomena such as magnetism. Certain semiconductors and 2D materials possess a similar property known as the "valley" degree of freedom. In these materials, the states at the Fermi surface are distributed in pockets, in the crystal momentum space. Each of these pockets is called a valley, which corresponds to a specific electronic state. Graphene is a zero-gap semiconductor with two valleys (usually called K and K'); its conduction and valence bands meet at discrete points, known as Dirac points.

In the valley
Just as "spintronics" – which makes use of the internal degree of freedom of spin to store and manipulate bits of information – this valley degree of freedom is at the centre of a stream of research aiming to exploit it for "valleytronics", where the information would be stored as discrete values of

Luis Foa Torres leads a team at the University of Chile, working on the electronic and optoelectronic properties of nanomaterials and nanodevices. www.foa-torres.com, e-mail: lfoa@u.uchile.cl
Sergio O Valenzuela is an ICREA research professor at the Catalan Institute of Nanoscience and Nanotechnology in Barcelona. Spain. www.valenzuela.com, e-mail: sfo@icrea.cat



Change of tack

Why Nobel laureates love to leave the herd

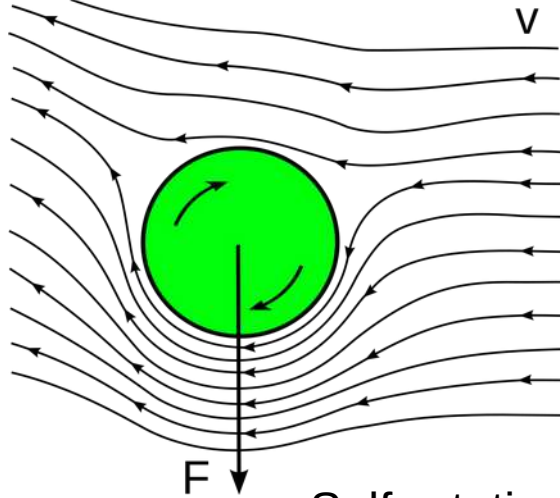


Emergence of graphene
Developing a low-cost, lab-grade microscope that examines carbon
How physics graduates should prepare for work
Valley of opportunities
Exploring graphene's perplexing properties

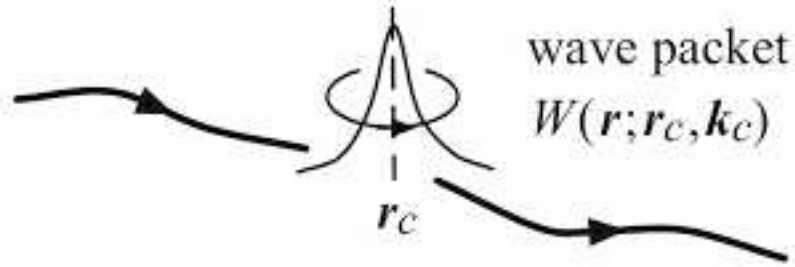
Connection with the Berry phase

Berry curvature and **self-rotation** of the wave-packets in certain **special** crystals.

Image: Wikimedia commons



Self-rotation enables **transverse motion** as in the Magnus effect.



Outline of this talk

Back in 1879: The Hall effect

The integer Quantum Hall effect

Topological connections

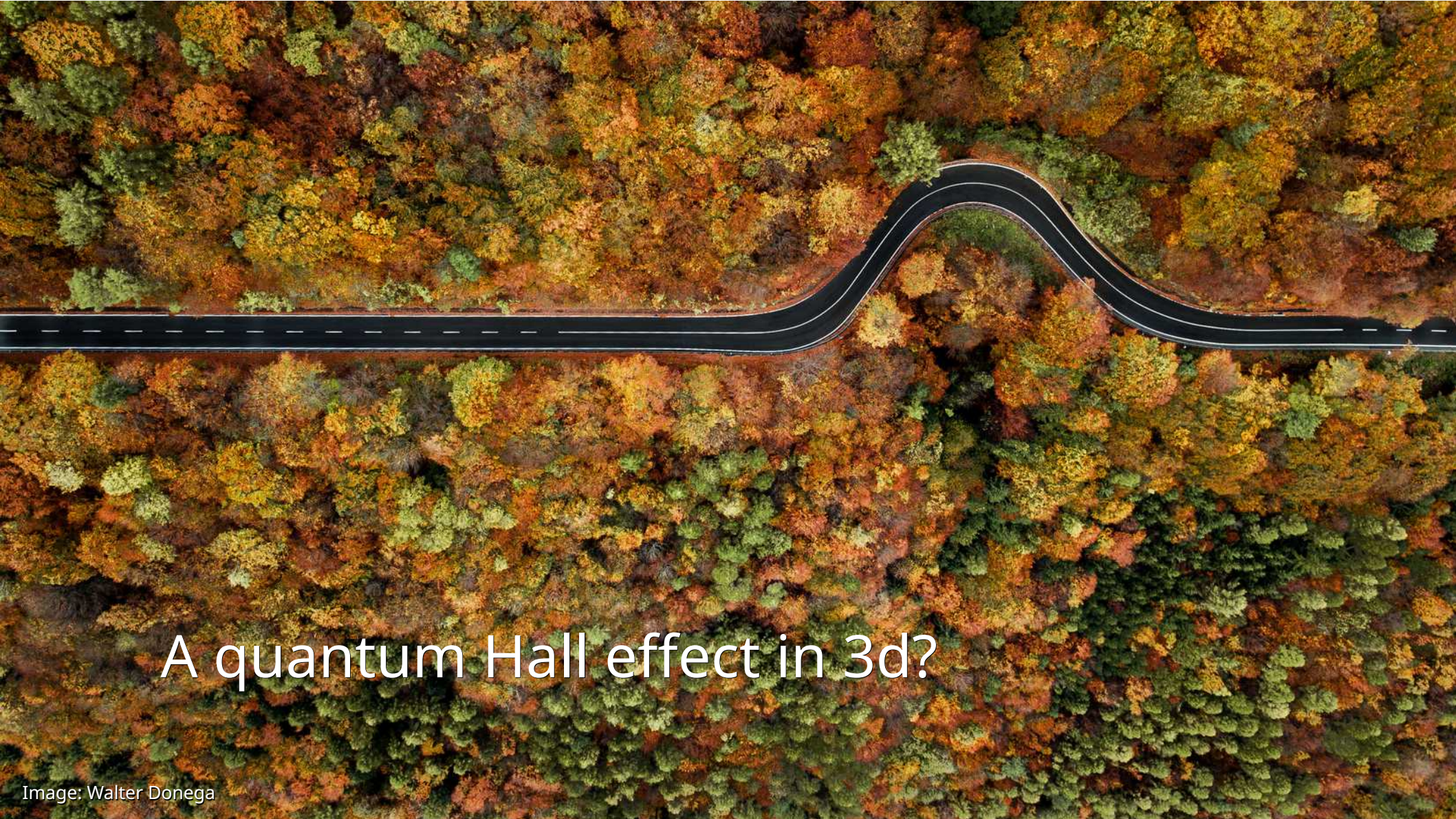
The anomalous Quantum Hall effect

The Quantum Spin Hall effect

The **3D** Quantum Hall effect

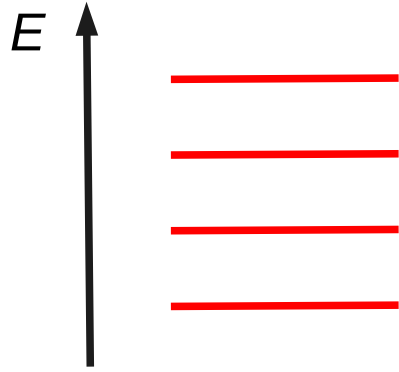
Light-induced Quantum Hall effect

Final remarks



A quantum Hall effect in 3d?

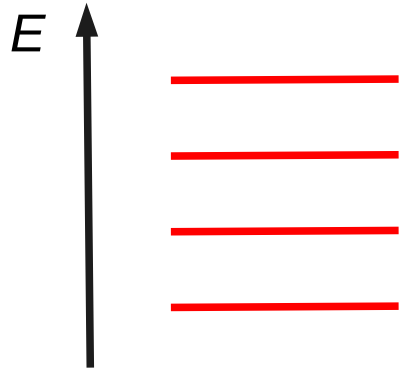
A Quantum Hall effect in 3d?



Landau levels for the “bulk” in **two-dimensions**

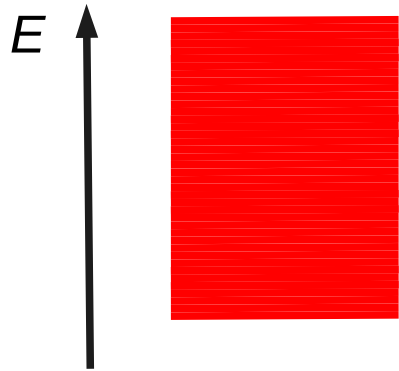
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

A Quantum Hall effect in 3d?



Landau levels for the “bulk” in **two-dimensions**

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega_c$$



The gap is lost in 3D !

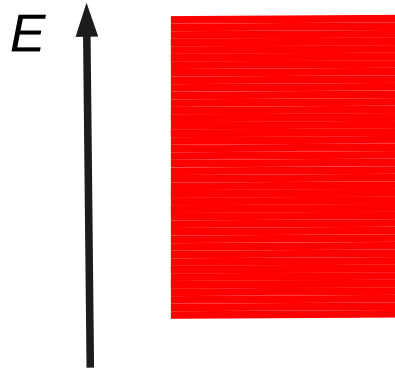
A Quantum Hall effect in 3d?

Proc. 18th Int. Conf. on Low Temperature Physics, Kyoto, 1987
Japanese Journal of Applied Physics, Vol. 26 (1987) Supplement 26-3

B. I. Halperin, "Possible states for a three-dimensional electron gas in a strong magnetic field," Jpn J. Appl. Phys. 26, 1913 (1987)

Possible States for a Three-Dimensional Electron Gas in a Strong Magnetic Field

Bertrand I. HALPERIN



The gap is lost in 3D.

Unless some intrinsic instability opens a gap.

Halperin predicted that in such case the Hall resistance would take a value

period of the potential along **B** $\lambda h/e^2$

A Quantum Hall effect in 3d?

Proc. 18th Int. Conf. on Low Temperature Physics, Kyoto, 1987
Japanese Journal of Applied Physics, Vol. 26 (1987) Supplement 26-3

Possible States for a Three-Dimensional Electron Gas in a Strong Magnetic Field

Bertrand I. HALPERIN

LETTER

<https://doi.org/10.1038/s41586-019-1180-9>

van der Waals layered material

Three-dimensional quantum Hall effect and metal–insulator transition in ZrTe_5

Fangdong Tang^{1,8}, Yafei Ren^{2,8}, Peipei Wang¹, Ruidan Zhong³, John Schneeloch³, Shengyuan A. Yang^{4*}, Kun Yang^{5,6}, Patrick A. Lee⁷, Genda Gu³, Zhenhua Qiao^{2*} & Liyuan Zhang^{1*}

F. Tang et al., “Three-dimensional quantum Hall effect and metal–insulator transition in ZrTe_5 ,” Nature **569**, 537 (2019).

A Quantum Hall effect in 3d?

Featured in Physics

Editors' Suggestion

Theory for the Charge-Density-Wave Mechanism of 3D Quantum Hall Effect

Fang Qin (覃昉), Shuai Li, Z. Z. Du, C. M. Wang, Wenqing Zhang, Dapeng Yu, Hai-Zhou Lu, and X. C. Xie
Phys. Rev. Lett. **125**, 206601 – Published 9 November 2020

Physics See Viewpoint: [Digging into the 3D Quantum Hall Effect](#)

Physics

VIEWPOINT

Digging into the 3D Quantum Hall Effect

Theorists invoke electron-phonon interactions to explain the recent observation of the quantum Hall effect in a 3D electronic system.

By Luis E. F. Foa Torres

<https://physics.aps.org/articles/v13/170>

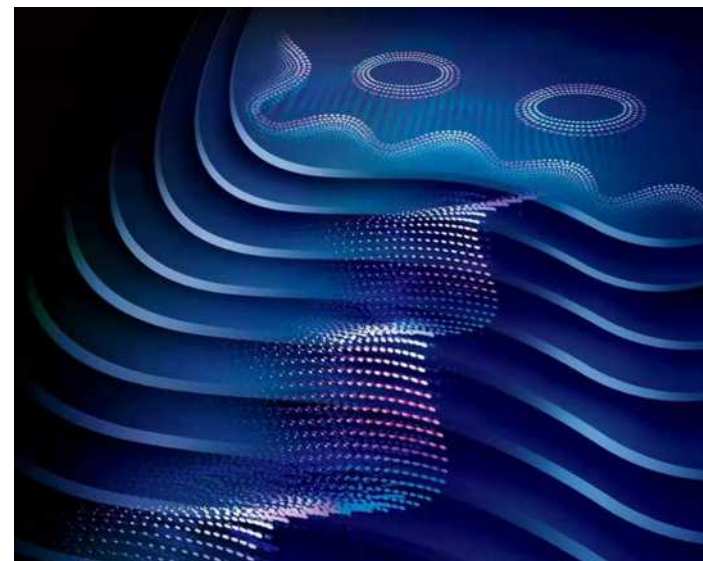


Image: Wang Guoyan & He Cong

An aerial photograph of a dark asphalt road that winds through a dense forest. The trees are in various stages of autumn, showing shades of red, orange, yellow, and some green. The road has white dashed lines and curves in a series of S-shapes. The sky is not visible.

Outline of this talk

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Light-induced quantum Hall effect

LETTERS

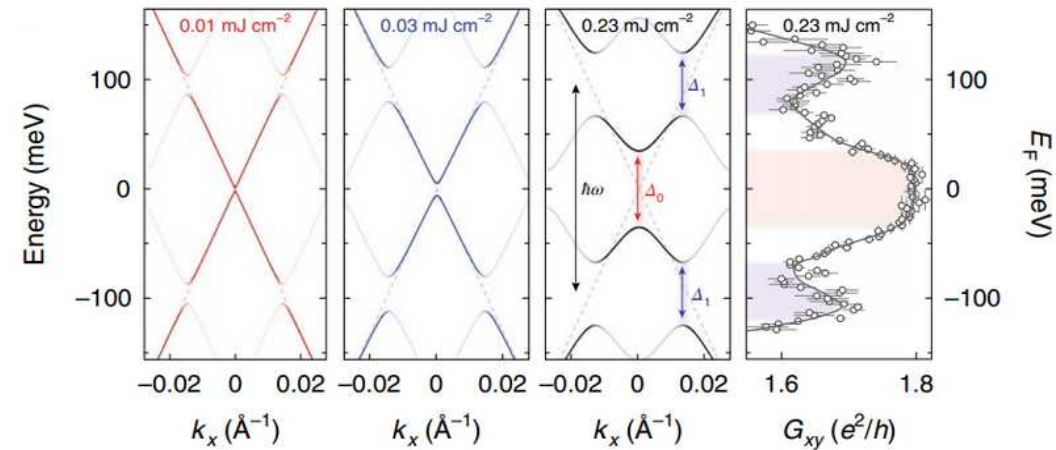
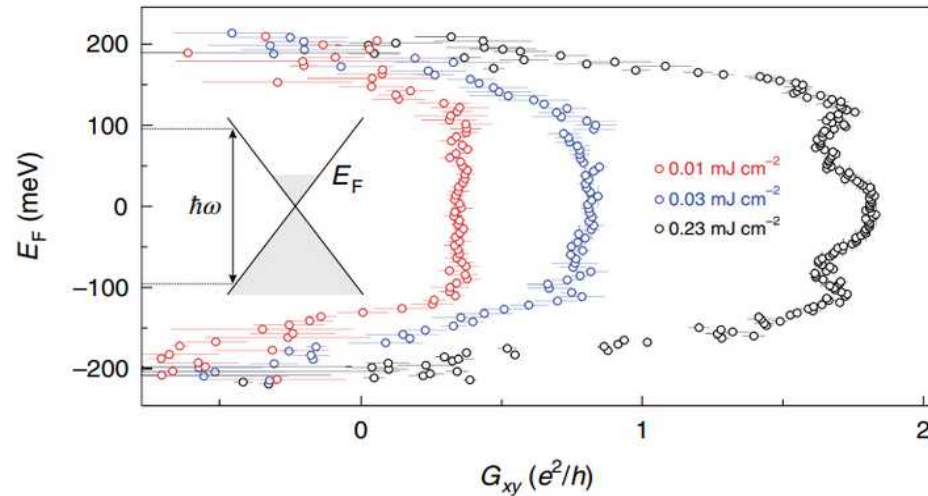
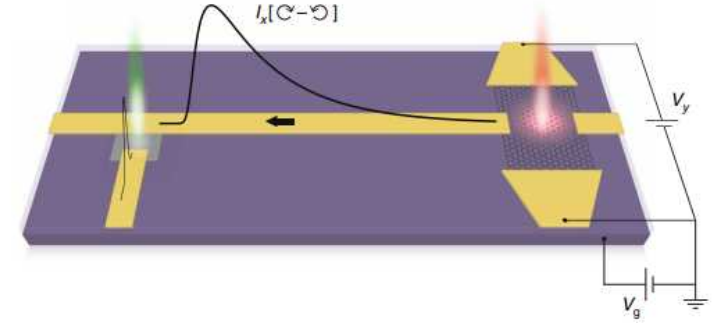
<https://doi.org/10.1038/s41567-019-0698-y>

nature
physics

Nature Physics **16**, 38 (2020)

Light-induced anomalous Hall effect in graphene

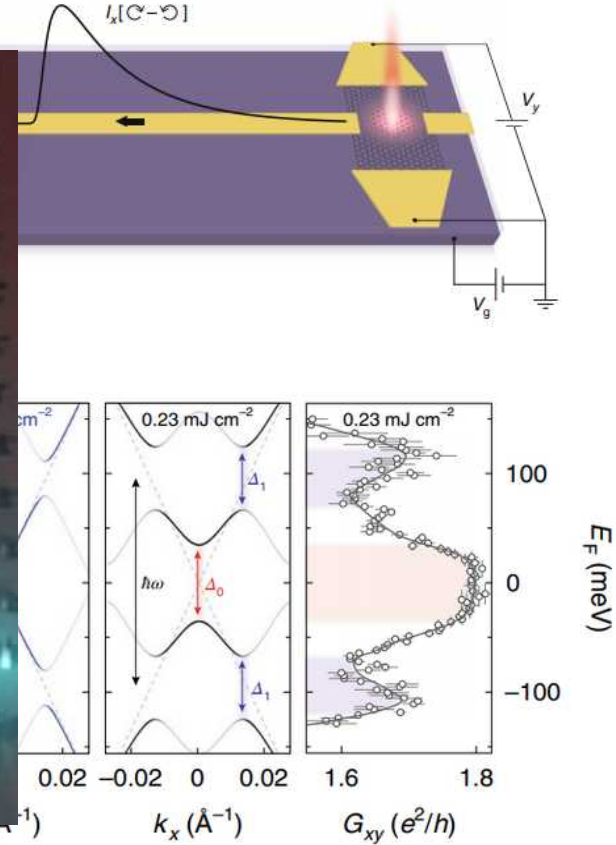
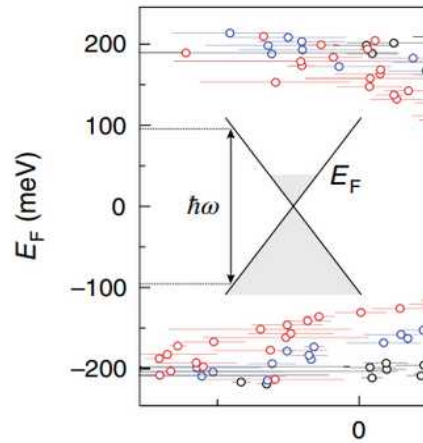
J. W. McIver^{1,3*}, B. Schulte^{1,3}, F.-U. Stein^{1,3}, T. Matsuyama¹, G. Jotzu¹, G. Meier¹ and A. Cavalleri^{1,2*}



LETTERS

nature
physics

J. W. McIver^{1,3*}, B. Schulte^{1,3}, F.-U. St



Light-induced quantum Hall effect

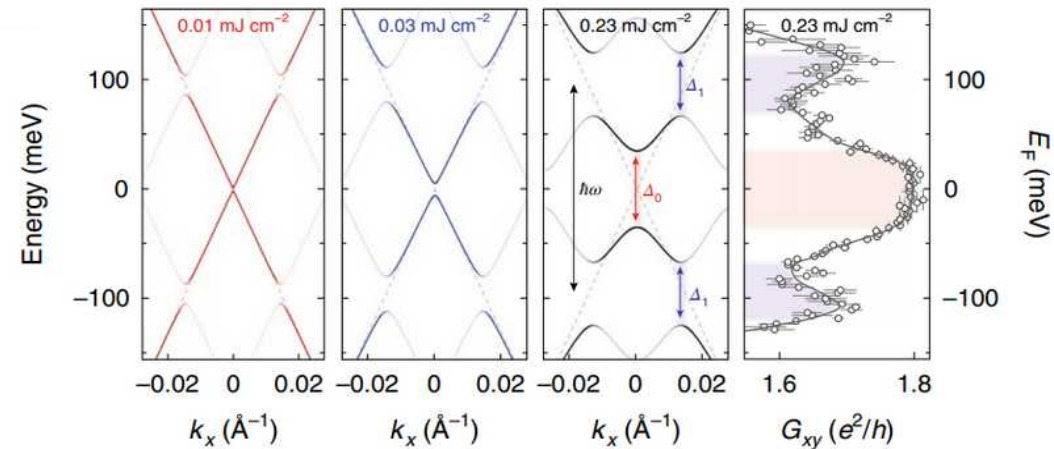
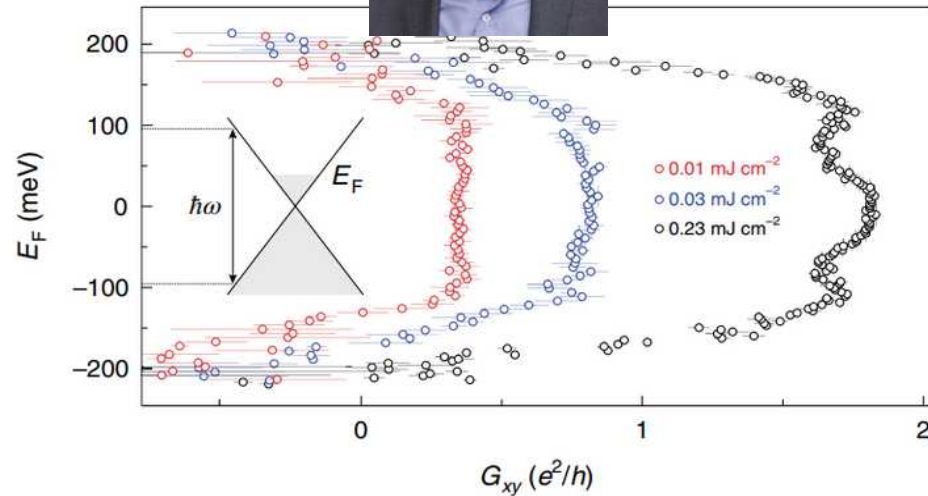
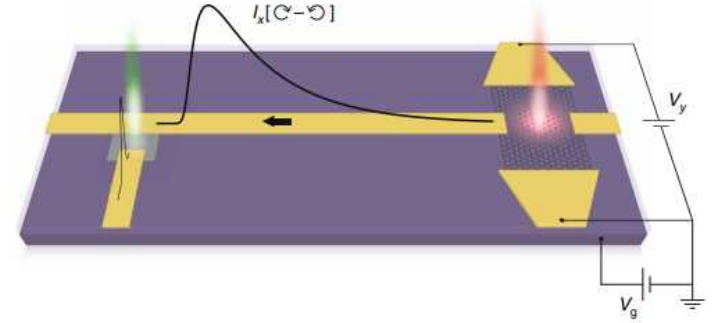
LETTERS

<https://doi.org/10.1038/s41567-019-0698-y>

nature
physics

Light-induced quantum Hall effect in graphene

J. W. McIver^{1,3*}, B. Schulte^{1,3}, F. ...
ama¹, G. Jotzu¹, G. Meier¹ and A. Cavalleri^{1,2*}



Final remarks

Quantum Hall effects



artificial systems
under
extreme conditions



materials
and
applications



Quantum Hall effects

PHYSICAL REVIEW LETTERS **122**, 200001 (2019)

Essay: Quantum Hall Effect and the New International System of Units

Klaus von Klitzing

Max Planck Institut für Festkörperforschung, Heisenbergstraße 1, D-70569 Stuttgart, Germany



(Received 18 March 2019; published 20 May 2019)

On 20 May 2019 the International System of Units will switch over to a new, globally accepted set of definitions based on constants of nature. The quantum Hall effect, discovered 39 years ago, plays an essential role in this development and will now contribute to a new definition of the kilogram. Ironically, the title of publication in *Physical Review Letters* that introduced the quantum Hall effect is, in hindsight, inaccurate in view of the revised units [K. von Klitzing, G. Dorda, and M. Pepper, New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance, *Phys. Rev. Lett.* **45**, 494 (1980)].

DOI: 10.1103/PhysRevLett.122.200001



materials
and
applications



Final remarks

Quantum Hall effects



THE SI

The SI — the modern metric system — has seven base units from which all other measurement units can be derived. On May 20, 2019, four of them — the kilogram, kelvin, ampere and mole — were redefined in terms of constants of nature. The remaining three — the second, meter, and candela — are already based on universal constants.

Click on the SI symbols below for more information.



materials
and
applications



<https://www.nist.gov/si-redefinition>

Quantum Hall effects

[Submitted on 20 May
2021]
arXiv:2105.09954 [cond-mat.mtrl-
sci]

All Topological Bands of All Stoichiometric Materials

Maia G. Vergniory,^{1,2,*} Benjamin J. Wieder,^{3,4,5,*} Luis Elcoro,⁶ Stuart S. P.
Parkin,⁷ Claudia Felser,⁸ B. Andrei Bernevig,⁵ and Nicolas Regnault^{5,9,†}

38,298 unique materials (nonmagnetic)

52.65% of all materials are topological at EF



materials
and
applications



Quantum Hall effects



Macroscopic quantum phenomena

Quantum Hall effects



Macroscopic quantum phenomena



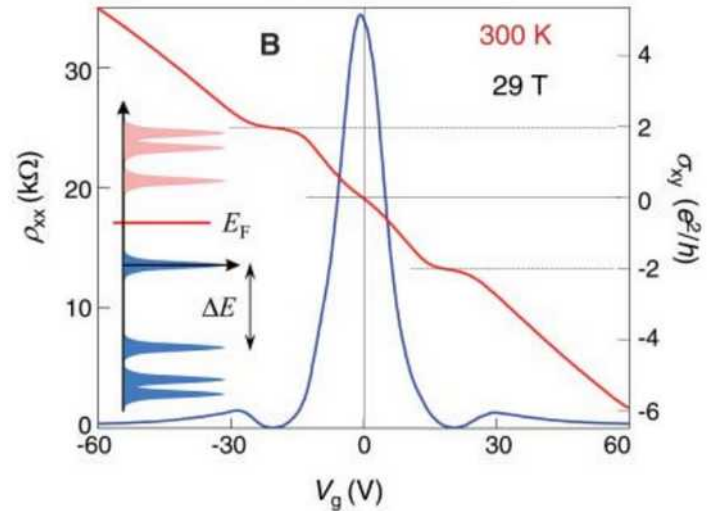
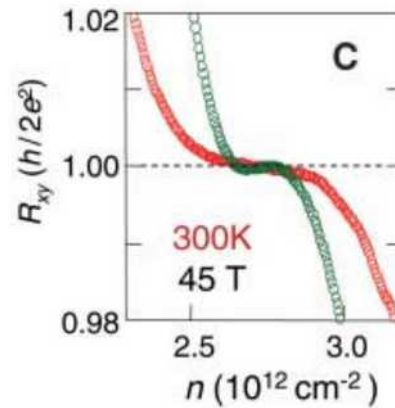
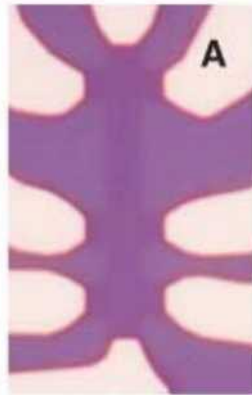
¡Gracias!

Landau levels: The case of graphene

Room-Temperature Quantum Hall Effect in Graphene

K. S. Novoselov,¹ Z. Jiang,^{2,3} Y. Zhang,² S. V. Morozov,¹ H. L. Stormer,² U. Zeitler,⁴ J. C. Maan,⁴
G. S. Boebinger,³ P. Kim,^{2*} A. K. Geim^{1*}

Science 09 Mar 2007:
Vol. 315, Issue 5817, pp. 1379
DOI: 10.1126/science.1137201



$$E_N = v_F \sqrt{|2e\hbar BN|}$$

General features:

Halperin's argument: propagating edge states require the presence of extended states so as to stop the spectral flow.

B. I. Halperin, "Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential," Physical Review B 25, 2185 (1982).

Semiclassical picture: internal rotation of the wavepacket



Precise Quantization of the Anomalous Hall Effect near Zero Magnetic Field

A. J. Bestwick,^{1,2} E. J. Fox,^{1,2} Xufeng Kou,³ Lei Pan,³ Kang L. Wang,³ and D. Goldhaber-Gordon^{1,2,*}

¹*Department of Physics, Stanford University, Stanford, California 94305, USA*

²*Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory,
2575 Sand Hill Road, Menlo Park, California 94025, USA*

³*Department of Electrical Engineering, University of California, Los Angeles, California 90095, USA*

(Received 16 January 2015; revised manuscript received 16 March 2015; published 4 May 2015)

We report a nearly ideal quantum anomalous Hall effect in a three-dimensional topological insulator thin film with ferromagnetic doping. Near zero applied magnetic field we measure exact quantization in the Hall resistance to within a part per 10 000 and a longitudinal resistivity under $1\ \Omega$ per square, with chiral edge transport explicitly confirmed by nonlocal measurements. Deviations from this behavior are found to be caused by thermally activated carriers, as indicated by an Arrhenius law temperature dependence. Using the deviations as a thermometer, we demonstrate an unexpected magnetocaloric effect and use it to reach near-perfect quantization by cooling the sample below the dilution refrigerator base temperature in a process approximating adiabatic demagnetization refrigeration.