Mono-parametric quantum charge pumping:
Interplay between spatial interference and photon-assisted tunneling

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We analyze quantum charge pumping in an open ring with a dot embedded in one of its arms. We show that cyclic driving of the dot levels by a *single* parameter leads to a pumped current when a static magnetic flux is simultaneously applied to the ring. Based on the computation of the Floquet-Green's functions, we show that for low driving frequencies ω_0, the interplay between the spatial interference through the ring plus photon-assisted tunneling gives an average direct current, which is proportional to ω_0^2. The direction of the pumped current can be reversed by changing the applied magnetic field.

I. INTRODUCTION

A direct current (dc) is usually associated to a dissipative flow of the electrons in response to an applied bias voltage. However, in systems of mesoscopic scale a dc current can be generated even at *zero* bias. This captivating quantum coherent effect is called *quantum charge pumping*[1–3] and it is of considerable interest both theoretically[1–8] and experimentally.[9,10] A device capable of providing such effect is called a *quantum pump* and typically involves the cyclic change of two device-control parameters with a frequency ω_0. The operational regime of the pump can be characterized according to the relative magnitude between ω_0 and the inverse of the time taken for an electron to traverse the sample, $1/\tau_T$. When $\omega_0 \ll 1/\tau_T$ the pump is in the so-called adiabatic regime, whereas the opposite case, $\omega_0 \gg 1/\tau_T$, the pump is in the nonadiabatic regime.

For adiabatic pumping, Brouwer[3] gave an appealing approach that is based on a scattering matrix formulation to low-frequency ac transport due to Büttiker *et al.*[11] In this formulation, the pumped current, which flows in response to a the cyclic variation of a set $\{X_j\}$ of device-control parameters, is expressed in terms of the scattering matrix $S(\{X_j\})$ of the system. One of the outcomes of this parametric pumping theory, which is valid in the low-frequency regime ($\omega_0 \ll 1/\tau_T$) and up to first order in frequency, is that the charge pumped during a cycle is proportional to the area enclosed by the path in the scattering matrix parameter space. Thus, to have a nonvanishing pumped charge, at least *two* time-dependent parameters that oscillate with a frequency ω_0 and with a nonvanishing phase difference φ between them are needed.

In this context, a natural question that arises is whether a pumped current can be obtained using a *single* time-dependent parameter. In most of the works considered up to now, at least *two* parameters are used to obtain pumping. A typical configuration that has been extensively studied theoretically and experimentally[10] consists of a dot connected to two leads with two out-of-phase time-dependent gate voltages that produce cyclic changes in its shape [see Fig. 1(a)]. In contrast, pumps based on a *single* parameter variation have attracted much less attention. This is partly due to the

fact that no pumping can be obtained from them in the low-frequency regime up to first order in ω_0. Hence, obtaining a nontrivial result requires going beyond the adiabatic limit described by the standard parametric pumping theory[3] as in Refs. 5, 12, and 13. In spite of giving a current, which, at low frequencies, is *a priori* weaker than the one obtained using a two-parameter variation, they can give comparable pumped currents at intermediate and high frequencies.[14] Besides, the understanding of such "mono-parametric pumps" constitutes a necessary step in the comprehension of driven systems.

Previous theoretical studies in this direction include the works by Kravtsov and Yudson[16] and Aronov and Kravtsov,[17] where pumping in a ring (not connected to leads) threaded by a time-dependent flux was studied. In Ref. 18, Wang *et al.* considered the case in which the height of one of the barriers of a double-barrier system connected to external leads is modulated periodically. This modulation *dynamically* breaks the inversion symmetry of the system producing a pumped current. Other theoretical works aiming at the fre-
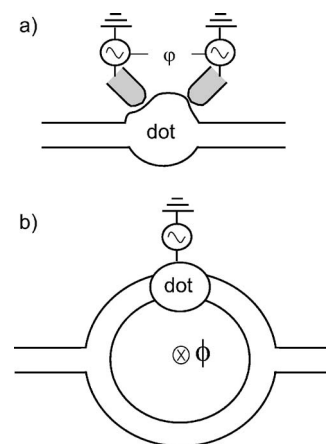

FIG. 1. (a) Schematic representation of a typical quantum pump consisting in an open dot driven by two out-of-phase time-dependent gate voltages. (b) Scheme of the system considered in this work, a ring connected to two leads. The ring, which is threaded by a magnetic flux, contains a dot embedded in one of its arms. Charge pumping is obtained by driving the dot levels through a time-periodic potential.

quency dependence of the pumped current for situations beyond the adiabatic approximation were also reported in Refs. 12 and 13. On the other hand, experimental studies using a single-parameter modulation have been reported recently,[19,20] showing predominant rectification effects[15] at low frequencies and quantum pumping in the high-frequency regime.

In this work we focus in mono-parametric pumping in systems connected to external leads. Specifically, we study a quantum pump consisting of a ring with a dot that is subjected to a *single* time-periodic gate voltage embedded in one of its arms, as represented in Fig. 1(b).[21] The ring is threaded by a *static* magnetic field, which produces the left-right symmetry breaking needed for pumping. The unique role of the time-dependent parameter in this pump is to provide for photon-assisted channels. We show that in the low-frequency regime ($\omega_0 \ll 1/\tau_T$) as a result of the interplay between spatial interference through the ring and photon-assisted processes, this device produces a dc current that is proportional to ω_0^2 and whose direction can be reversed by tuning the applied magnetic field.

Our theoretical framework mostly follows Ref. 22 and is based on the use of Floquet's theory[23,24] to write the average current in terms of the Fourier components of the retarded Green's functions for the system. However, instead of solving an eigenvalue problem as in Refs. 22 and 25, we completely rely on the computation of the Floquet-Green's functions for the system. The resulting picture is that of an equivalent time-independent problem in a higher dimensional space (similar to the one previously obtained for electron-phonon interactions[26,27]) and is specially suited for discrete Hamiltonians offering, thus, a promising application to molecular systems.[28]

This work is organized as follows. First, we briefly introduce our theoretical tools and present the case of a driven double barrier as an example to motivate the subsequent discussion of mono-parametric quantum pumping. Then we focus in our model system, discussing our analytical and numerical results.

II. THEORY

In this section, we introduce the theoretical tools that will be used to address our specific problem. In order to keep the discussion general, we consider at this point a generic system consisting of a sample region that is connected to two leads (left and right). The time-dependent Hamiltonian can be written in the form

$$H = H_{\text{sample}}(t) + H_{\text{leads}} + H_{\text{contacts}},$$

where the terms correspond to the contributions from the sample, the leads, and the sample-leads coupling, respectively. Following Ref. 22, we focus on the regime of quantum coherent transport, and electron-electron interactions are not considered.

We consider a situation in which both leads are in thermal equilibrium with a common chemical potential, i.e., $f_L(\varepsilon) = f_R(\varepsilon) \equiv f(\varepsilon)$ for all energies. Then, the current averaged over one period of the modulation is given by[22,25]

$$\bar{I} = \frac{e}{h} \sum_n \int d\varepsilon [T_{R \leftarrow L}^{(n)}(\varepsilon) - T_{L \leftarrow R}^{(n)}(\varepsilon)] f(\varepsilon), \tag{1}$$

where

$$T_{R \leftarrow L}^{(n)}(\varepsilon) = 2\Gamma_R(\varepsilon + n\hbar\omega_0)|G_{RL}^{(n)}(\varepsilon)|^2 2\Gamma_L(\varepsilon) \tag{2}$$

are the transmission probabilities for an electron with energy ε from left to right involving the absorption (or emission) of an energy $n\hbar\omega_0$ [and similarly for $T_{L \leftarrow R}^{(n)}(\varepsilon)$]. Here $G^{(n)}(\varepsilon) = (1/T) \int_0^T dt e^{-in\omega_0 t} G(t, \varepsilon)$ are the coefficients of the Fourier decomposition of the retarded Green's function $G^R(t, \varepsilon)$, and $\Gamma_{L(R)}(\varepsilon)$ are given by the imaginary part of the retarded self-energy correction due to the corresponding lead, $\Gamma_{L(R)}(\varepsilon) = -\text{Im}\,\Sigma_{L(R)}(\varepsilon)$.

Equation (1) was derived in Ref. 22 by solving the Heisenberg equations of motion for the creation and annihilation operators and taking advantage of the time periodicity of the Hamiltonian. The transmission probabilities $T_{\alpha \leftarrow \beta}^{(n)}(\varepsilon)$ were expressed in terms of the Fourier components $G_{\beta\alpha}^{(n)}(\varepsilon)$ of the retarded Green's functions. This procedure is, for non-interacting electrons, formally equivalent to the use of the Keldysh formalism[29,30] but in contrast to previous works along that path, the time periodicity of the Hamiltonian is exploited through the use of Floquet's theory.[31]

In Ref. 22 the Fourier components $G_{\beta\alpha}^{(n)}(\varepsilon)$ of the retarded Green's functions were written (after tracing over the degrees of freedom in the leads) in terms of the solutions of Floquet's equation[23,24] for the sample region. Although the Floquet's states and their corresponding quasienergies can be obtained numerically, this can take a significant computational power depending on the system size. Here, we use instead a different strategy: the essential idea is to write Eq. (1) completely in terms of the Floquet-Green's functions.

To such end we note that the Fourier coefficients $G^{(n)}(\varepsilon)$ can be written as (see Appendix)

$$G_{\alpha\beta}^{(n)}(\varepsilon) = G_{(\alpha,n),(\beta,0)}^F(\varepsilon), \tag{3}$$

where the Floquet-Green's functions

$$G_{(\alpha,n),(\beta,0)}^F \equiv \langle \alpha, n | (\varepsilon I - H_F)^{-1} | \beta, 0 \rangle \tag{4}$$

are defined in terms of the Floquet Hamiltonian

$$H_F = H(t) - i\hbar \frac{\partial}{\partial t}. \tag{5}$$

Note that both H_F and G^F are defined in the composed Hilbert space $R \otimes T$, where R is the space of functions in real space and T is the space of periodic functions with period $\tau = 2\pi/\omega_0$. The space T is spanned by the set of orthonormal Fourier vectors $\langle t | n \rangle \equiv \exp(in\omega_0 t)$, where n is an integer. A suitable basis for this so called *Floquet or Sambe space*,[24] $R \otimes T$, is thus given by $\{|i, n\rangle \equiv |i\rangle \otimes |n\rangle\}$, where $|i\rangle$ corresponds to a state localized at site i.

Substituting Eq. (3) in Eqs. (2) and (1), we write the average current completely in terms of the Floquet-Green's functions G^F (see Appendix). The key point here is that this renders an equivalent time-independent problem in a higher-dimensional space, $R \otimes T$. Therefore, the full power of the
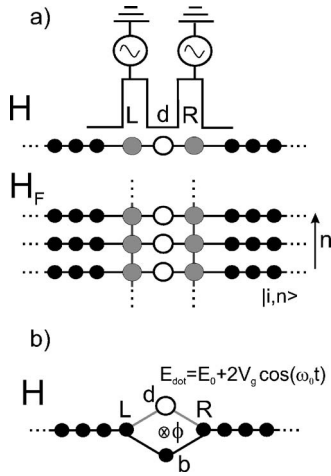
FIG. 2. (a) Top: Scheme of a double-barrier system driven by time-dependent gate voltages applied to each of the barriers. Bottom: Representation of the Floquet Hamiltonian corresponding to the tight-binding model introduced in the text for the system shown in the top. Circles correspond to different states $|i,n\rangle$ in Floquet space, and lines are off-diagonal matrix elements. States along a vertical line correspond to the same spatial states with different number n of photon quanta. Note that the phase of the vertical matrix elements connecting different Floquet states generally depends on the direction of the transition. For example, the matrix element of the Floquet Hamiltonian connecting the $|L,n\rangle$ state with $|L,n+1\rangle$ is $V_{gL}\exp(i\varphi_L)$, whereas the matrix element for the reverse process is $V_{gL}\exp(-i\varphi_L)$. (b) Representation of the model Hamiltonian used in the text for the situation in Fig. 1(b).

recursive Green's functions techniques[32] can be used accordingly.

To compute these functions we consider only the Floquet states $|j,n\rangle$ within some range for n, i.e., $|n|\leqslant N_{max}$. This range can be successively expanded until the answer converges, giving thus a variational (nonperturbative) method. The resulting scheme is similar to the one introduced in Refs. 26 and 27 for the problem of phonon-assisted electron transport.[35] The main differences[36] are that (i) for phonons the temperature enters naturally in the population of the different channels, (ii) the phonon spectrum is bounded from below, and (iii) the matrix element for phonon emission and absorption depend on the number N of phonons present in the system before the scattering process.

In order to fix ideas and motivate the subsequent discussion, let us see how this picture works for the case of a driven double barrier and then we turn to the study of our model system.

A. Example: Driven double barrier

Let us consider a system as depicted in the top of Fig. 2(a). A simple Hamiltonian for that situation is given by

$$H_{sample} = \widetilde{E}_L(t)c_L^+c_L + E_0 c_d^+ c_d + \widetilde{E}_R(t)c_R^+ c_R + V_{L,d}(c_L^+ c_d + \text{H.c.})$$
$$+ V_{R,d}(c_R^+ c_d + \text{H.c.}).$$

where c_α^+ (c_α) are the creation (destruction) operators at site

α and $\widetilde{E}_L = E_L + 2V_{gL}\cos(\omega_0 t + \varphi_L)$ and $\widetilde{E}_R = E_R + 2V_{gR}\cos(\omega_0 t + \varphi_R)$ are the energies of the barrier sites, which are modulated periodically. Only one energy state E_0 inside the double barrier is considered. The leads are regarded as one-dimensional tight-binding chains, which are coupled to the sites L and R in the sample as shown in Fig. 2(a).

The Floquet Hamiltonian [see Eq. (5)] for this system in the composed space $R\otimes T$ is represented in the bottom of Fig. 2(a). The horizontal dimension corresponds to states localized in different spatial positions $|i\rangle$, whereas the vertical one corresponds to different Fourier states $|n\rangle$. Lines along the horizontal direction correspond to hoppings between states localized in different positions, whereas the vertical ones are determined by the Fourier coefficients of the time-dependent part of the Hamiltonian. Thus, we can clearly see that the resulting picture is that of a higher-dimensional time-independent system. The transport properties can be computed directly from the Green's functions in this space as discussed before.

Using this scheme, we can now try to understand the main difference between a one-parameter and a two-parameter pump. Symmetry breaking is at the heart of quantum charge pumping: to obtain a directed current at zero-bias potential, the left-right symmetry (LRS) of the system must be broken. This can be achieved by breaking either time-reversal symmetry (TRS) or inversion symmetry (IS). However, breaking of the LRS alone does not guarantee a nonvanishing pumped current. In a two-terminal configuration, for example, even when these symmetries are statically broken (i.e., through a time-independent potential), TRS under magnetic-field inversion and unitarity of the scattering matrix assures that the transmittance is insensitive to the direction of propagation,[37] i.e., $T_\rightarrow(\varepsilon) = T_\leftarrow(\varepsilon)$, and hence there is no pumped current. When a time-dependent potential is added, photon-assisted processes come into play opening new paths for transport. The resulting picture is that of a multichannel system, and the reciprocity relation $T_\rightarrow(\varepsilon) = T_\leftarrow(\varepsilon)$ valid for the static case is replaced by an integral relation, $\int_0^\infty T_\rightarrow(\varepsilon)d\varepsilon = \int_0^\infty T_\leftarrow(\varepsilon)d\varepsilon$, thus allowing for a nonvanishing pumping current. The crucial difference between the situation where only one time-dependent parameter is present and the one with two, is the possibility of making a closed loop in Floquet space involving at least two vertical processes (or two "paths" in Floquet space). When the two-parameter variation is out of phase $\varphi_L - \varphi_R \neq 0$, *there will be a nonvanishing accumulated phase through the loop in a way that is analogous to a magnetic flux.* For $\varphi_L - \varphi_R \neq 0 \mod(\pi)$ the accumulated phase is different, depending on the direction of motion. Note that this directional asymmetry of the electronic motion (which is a consequence of a dynamical breaking of IS and TRS) is maximum when $\varphi_L - \varphi_R = \pi/2 \mod(\pi)$. We will see that for a system as shown in Fig. 1(b), where only one time-dependent gate voltage is present, the directional asymmetry is provided by the static magnetic field and is manifested as a pumped current only when photon-assisted processes are allowed.

III. MODEL

In what follows we focus on a system as depicted in Fig. 1(b), consisting of a quantum dot embedded in an arm of a

ring, which, in turn, is connected to two leads (the dot is placed symmetrically between the leads). The ring is threaded by a magnetic flux ϕ. We do not consider the electron-electron interaction in the dot, which is a reasonable approximation for strong dot-ring coupling (open dot). We demonstrate our results using a lattice Hamiltonian similar to the one used in Ref. 33 and more recently in Ref. 34. Although we use the simplest model with this geometry, i.e., a four-sites ring, our results can be extended to more general situations involving, for example, several sites in each arm of the ring and arbitrary potential profile. Our Hamiltonian is depicted in Fig. 2(b).

The magnetic flux ϕ is introduced as a phase factor in $V_{L,d}=V_{d,L}^*=|V_{L,d}|\exp(i2\pi\phi/\phi_0)$ (note that gauge invariance allows one to place it on any bond of the ring), b and d are the labels used for the site in the reference arm of the ring and the dot, respectively. The energy of the state in the dot E_{dot}, which is modulated periodically by a gate voltage, is modeled through $E_{\text{dot}}=E_0+2V_g\cos(\omega_0 t)$. The leads are modeled as one-dimensional tight-binding chains with zero site energy and hopping matrix element V [$H_{\text{leads}}=\Sigma_{\langle i,j\rangle \text{ with } i,j\in l,r}V(c_i^+c_j+\text{H.c.})$].

IV. RESULTS AND DISCUSSION

Now we turn to the study of the pumping properties of our system. First, we write the kernel in Eq. (1) as a sum of contributions due to the different channels

$$\sum_n [T_{R\leftarrow L}^{(n)}(\varepsilon)-T_{L\leftarrow R}^{(n)}(\varepsilon)] \equiv \sum_n \delta T^{(n)}(\varepsilon) \equiv \delta T(\varepsilon), \quad (6)$$

where

$$\delta T^{(n)}(\varepsilon) = T_{R\leftarrow L}^{(n)}(\varepsilon)-T_{L\leftarrow R}^{(n)}(\varepsilon) = 2\Gamma_{(R,n)}(\varepsilon)$$
$$\times |G_{(R,n)\leftarrow(L,0)}^F(\varepsilon)|^2 2\Gamma_{(L,0)}(\varepsilon) - 2\Gamma_{(L,n)}(\varepsilon)$$
$$\times |G_{(L,n)\leftarrow(R,0)}^F(\varepsilon)|^2 2\Gamma_{(R,0)}(\varepsilon). \quad (7)$$

To gain an understanding of the physical mechanisms that give rise to pumping in our system, we compute the Floquet-Green's functions up to the first nonvanishing order in the driving amplitude. Using Dyson's equation for the Floquet-Green's functions, we obtain for the elastic component,

$$G_{(R,0)\leftarrow(L,0)}^F(\varepsilon) \simeq g_{R,L} + g_{R,d}V_g(g_{d,d}^{(+)}+g_{d,d}^{(-)})V_g g_{d,L}, \quad (8)$$

and similarly for $G_{(L,0)\leftarrow(R,0)}^F(\varepsilon)$. $g_{i,j}$ are the exact retarded Green's functions for the system in the absence of the time-dependent potential. The superscripts are a short notation to indicate that the corresponding Green's functions are evaluated at a displaced energy, $g_{i,j}^{(\pm)}\equiv g_{i,j}(\varepsilon\pm\hbar\omega)$; all the other functions are evaluated at the energy ε.

From Eq. (8), we can appreciate that the elastic component of the Floquet-Green's function connecting the left and right electrodes is the sum of two terms: one that corresponds to direct transmission from left to right and other that involves virtual photon emission and absorption in the dot. Thus, the modulus squared of $G_{(\alpha,0);(\beta,0)}^F(\varepsilon)$ contains three terms: (a) $|g_{\alpha,\beta}|^2$, which is ω independent and does not con-

tribute to the pumped current because it obeys the symmetry $|g_{\alpha,\beta}|=|g_{\beta,\alpha}|$.[37] (b) The next contribution is the modulus squared of the second term in Eq. (8), which is fourth order in the driving amplitude and can be neglected in a first approximation. (c) The last contribution is an interference term between the quantum mechanical amplitudes corresponding to direct tunneling and tunneling plus virtual photon emission and absorption. This term, which critically depends on the phase difference between the two terms in Eq. (8), has a directional asymmetry due to the presence of the magnetic field, which gives rise to a nonvanishing contribution to the pumped current.

This asymmetry is reflected in the fact that $|g_{L,d}|\neq|g_{d,L}|$ for $\phi\neq 0 \mod(\pi)$. To understand this difference, it is useful to note that $g_{L,d}$ (or $g_{d,L}$) is proportional to the effective hopping between the corresponding sites, namely, $\widetilde{V}_{L,d}$ (or $\widetilde{V}_{d,L}$, being the proportionality constant, the same for both cases). This effective hopping can be written as a sum of two terms, a direct one from site L to the dot and another that corresponds to the alternative path through the ring: $\widetilde{V}_{L,d}=V_{L,d}+V_{L,b}\widetilde{g}_b V_{b,R}\bar{g}_R V_{R,d}$ (a similar expression holds for $\widetilde{V}_{d,L}$), where $\widetilde{g}_b$ is the Green's function of the isolated b site renormalized by the presence of the R site and the right lead, $\bar{g}_R$ is the Green's function of the isolated right site renormalized by the right lead. It is easy to see that the interference between these two spatial paths is directionally asymmetric for $\phi\neq 0 \mod(\pi)$, giving, therefore, $|g_{L,d}|\neq|g_{d,L}|$. Again, we have a situation similar to the one in Eq. (8), but this time the interference takes place in real space.

A similar analysis can be performed based on the study of the Floquet-Green's functions involving a net photon absorption and emission

$$G_{(R,\pm1)\leftarrow(L,0)}^F(\varepsilon) \simeq g_{R,d}^{(\pm)}V_g g_{d,L}. \quad (9)$$

Again, the main observation is that the pumped current is originated from spatial interference through the ring plus the photon-assisted processes provided by the time-dependent variation of the dot's energy.

In order to obtain the frequency dependence of the pumped current, we assume the validity of the broadband approximation and expand the Green's functions $g_{\alpha\beta}^{(\pm)}=g_{\alpha\beta}(\varepsilon\pm\hbar\omega_0)$ for low frequencies. Using this expansion in Eqs. (8) and (9), it can be seen that the overall frequency-independent contribution to δT is zero and results from a cancellation between the elastic and the inelastic contributions. The contributions to $\delta T^{(+)}$ that are first order in $\hbar\omega_0$ cancel with the corresponding ones in $\delta T^{(-)}$. Inspection of Eq. (8) shows that the linear term in $\delta T^{(0)}$ also vanishes as a consequence of the symmetry between the sidebands of absorption and emission.[38] Hence, we observe that the first nonvanishing contribution to the average current $\bar{I}$ is proportional to $(V_g\omega_0)^2$.

It must be noted that the predicted frequency dependence of the pumped current $\bar{I}$ is in consistency with the general results presented in Refs. 12 and 13 for different systems. Here, we interpret our results within the framework intro-
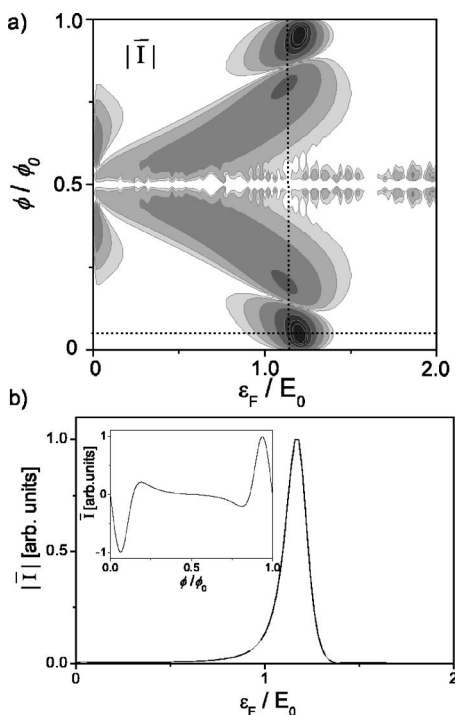
FIG. 3. (a) Contour plot of the the absolute value of the average dc current $\bar{I}$ for the system shown in Fig. 2(b). The horizontal scale corresponds to the Fermi energy in the leads, whereas the vertical axis is the flux through the ring. The values of the parameters in this plot are: $V_g=0.001$, $\hbar\omega=0.002$, $|V_{L,d}|=|V_{R,d}|=-0.5$; all the other hoppings are set equal to $V=-1$, which is taken as the unit of energy. Note the strong maximum for Fermi energies close to the dot's energy and flux $\phi\sim0.1\phi_0$. In (b) we show the dependence on the Fermi energy for $\phi=0.1\phi_0$ [the other parameters are the same as in (a)]. The inset in (b) shows the flux dependence close to the resonant point ($\varepsilon_F=1.15$). The plots in this lower panel correspond to traces along the dotted lines in the contour plot.

duced in Ref. 13: the charge pumped per cycle is determined not by the contour in parameter space, as in Ref. 3 (which in this case encloses a vanishing area), but by the contour in *phase space*[13] (which contains, in addition to the pumping parameters, their time-derivatives). The contour in phase space encompasses a nonvanishing area that is proportional to ω_0, giving thus the predicted quadratic frequency dependence for the pumped current $\bar{I}$.

The results obtained up to now are very general in the sense that they do not depend much on the specifics of the model as long as the geometry is preserved. Let us contrast these analytical results with the numerical results for arbitrary frequency and driving amplitude. The results shown in Figs. 3 and 4 are computed using Eqs. (1)–(3) for zero temperature. Satisfactory accuracy is obtained by considering Floquet's states with $|n|\leqslant4$.

A contour plot for the absolute value of the average current for small frequency and driving amplitudes is shown in Fig. 3(a); the horizontal axis corresponds to the position of the Fermi energy in the leads and the vertical one corresponds to the magnetic flux in units of the flux quantum. The regions with larger $\bar{I}$ values correspond to darker areas. We
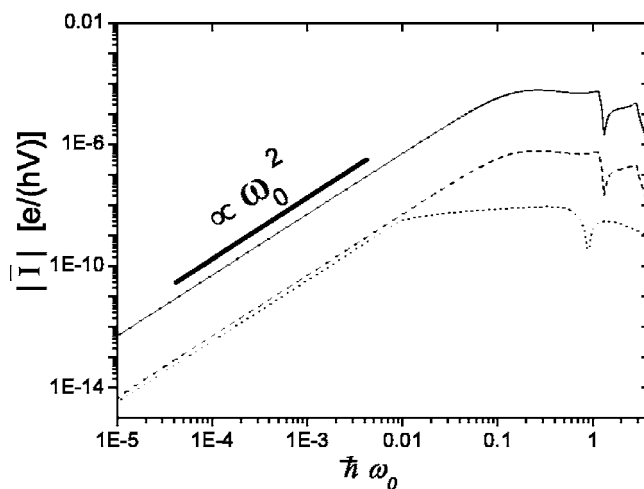

FIG. 4. Absolute value of the average dc current $\bar{I}$ for the system shown in Fig. 2(b) as a function of the driving frequency. The solid and dashed lines correspond to $V_g=0.01$ and $V_g=0.001$, respectively. The Fermi energy and the flux are chosen close to the resonant point ($\varepsilon_F=1.15$ and $\phi=0.1\phi_0$). The values of the other parameters are as in the previous figure. The dotted line corresponds to a small Fermi energy $\varepsilon_F=0.01$, $\phi=0.4\phi_0$, and $V_g=0.001$. Note that for this case (dotted line) the quadratic dependence with the driving frequency holds up to $\hbar\omega_0\sim\varepsilon_F$.

observe that the larger currents are located for energies close to the energy of the dot's level ($\varepsilon_F\sim1$) and small magnetic flux ($\phi\sim0.1\phi_0$). As the magnetic flux increases from zero to half flux quantum, the position of these maxima move due to interference inside the ring. At $\phi=0.5\phi_0$, the TRS of the system is restored and the pumped current vanishes.

In Fig. 3(b), we show the average current as a function of the Fermi energy for ($\phi\sim0.1\phi_0$). We observe a resonant behavior for Fermi energies close to the dot's energy. This is expected because photon-assisted processes are stronger close the resonant condition. In the inset of Fig. 3(b), we show the flux dependence near the resonant point ($\varepsilon_F=1.15$). The pumped current is periodic in the applied magnetic flux with a period equal to the flux quantum and several harmonics (up to the fifth) contribute importantly to this dependence. Interestingly, we can see that the pumped current can be reversed by tuning either the magnitude or the direction of the magnetic field.

Another interesting feature that we can appreciate in Fig. 3(a) is the appearance of very narrow maxima in the pumped current for Fermi energies of the order of the driving frequency [weak maxima close to the vertical $\varepsilon_F=0$ axis in Fig. 3(a)]. This is because, for energies smaller than $\hbar\omega_0$, the processes involving of photon emission are energetically forbidden, thus, generating a strong asymmetry between emission and absorption, which leads to a pumped current that decays in magnitude as the Fermi energy is increased. In this highly nonadiabatic situation, the previous theoretical analysis based on a low-frequency expansion of the Green's functions fails and the currents do not follow the predicted ω_0^2 dependence for $\hbar\omega_0\gtrsim\varepsilon_F$ (see dotted line in Fig. 4).

The pumped current as a function of $\hbar\omega_0$ is shown in Fig. 4. Different curves correspond to different values of the driv-

ing amplitude. A line corresponding to a quadratic behavior is also shown for reference. These results clearly show the predicted low-frequency dependence of the pumped current $\bar{I} \propto \omega_0^2$ up to frequencies of the order of the minimum between the width of the resonant level in the dot ($\Gamma_{dot} \sim 0.4$) and the Fermi energy. The dependence on the driving amplitude (not shown in the figure) also verifies a quadratic dependence up to moderate driving amplitudes ($V_g \sim 0.1V$).

V. CONCLUSIONS

In summary, we have studied quantum charge pumping in a system with a *single* time-periodic parameter using a formalism based on the computation of the Floquet-Green's functions. The resulting picture is that of a time-independent system in a higher-dimensional space where processes occurring in real space and photon-assisted processes enter in the same footing. This allows us to clarify the main differences between a one-parameter and a two-parameter pump.

Our pump consists of a ring connected to two leads and containing a "dot" embedded in one of its arms. The ring is threaded by a magnetic flux while the dot levels are subjected to a time-periodic gate voltage. We have shown that a pumped current proportional to the square of the driving frequency appears as a result of the combined effect of spatial interference through the ring and photon-assisted tunneling. The direction of the current can be changed by tuning either the direction or the magnitude of the magnetic field. It must be emphasized that the directional asymmetry needed to obtain quantum pumping is provided through the static magnetic field, whereas the unique role of the time-dependent parameter is to provide additional inelastic channels for transport. In this sense, a pumped current can be obtained using any other mechanism that provides such inelastic channels as long as the phase coherence of the composed system (sample plus inelastic scatterer) is preserved.

Note added in proof. Recently, we became aware of two related papers by L. Arrachea.[39]

ACKNOWLEDGMENTS

APPENDIX

In order to derive Eq. (3), we write the retarded Green's function $G_{\beta,\alpha}(t,\varepsilon)$ in terms of the time-evolution operator of the system $U_{\beta,\alpha}(t,t')$ [defined by the relations $|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$, $U(t_0,t_0) \equiv 1$]

$$G_{\beta,\alpha}(t,\varepsilon) = -\frac{i}{\hbar}\int_0^\infty d\tau \exp(i\varepsilon\tau/\hbar) U_{\beta,\alpha}(t,t-\tau). \quad (A1)$$

Then, using the well-known relation between the matrix elements of the time-evolution operator and the Floquet Hamiltonian[23]

$$\langle\beta|U(t,t_0)|\alpha\rangle = \sum_{n=-\infty}^{\infty} \langle\beta,n|\exp(-iH_F(t,t_0)/\hbar)|\alpha,0\rangle$$
$$\times \exp(in\omega t) \quad (A2)$$

in Eq. (A1) and integrating over τ, we obtain,

$$G_{\beta,\alpha}(t,\varepsilon) = \sum_{n=-\infty}^{\infty} \langle\beta,n|(\varepsilon 1 - H_F)^{-1}|\alpha,0\rangle\exp(in\omega t).$$
$$(A3)$$

The coefficients of the exponential can be identified as the Fourier coefficients in the Fourier expansion of $G_{\beta,\alpha}(t,\varepsilon) = \Sigma_n G_{\beta,\alpha}^{(n)}(\varepsilon)\exp(in\omega t)$ from where Eq. (3) follows.

Substituting this relation into Eqs. (2) and (1) gives the average current in terms of the Floquet-Green's functions

$$\bar{I} = \frac{e}{h}\sum_n \int d\varepsilon [T_{R\leftarrow L}^{(n)}(\varepsilon) - T_{L\leftarrow R}^{(n)}(\varepsilon)]f(\varepsilon), \quad (A4)$$

where

$$T_{R\leftarrow L}^{(n)}(\varepsilon) = 2\Gamma_{(R,n)}(\varepsilon)|G_{(R,n)\leftarrow(L,0)}^F(\varepsilon)|^2 2\Gamma_{(L,0)}(\varepsilon).$$

The transmittance in the reverse sense follows from the last equation by exchanging the L and R indexes.

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