

## Coherent versus Sequential Electron Tunneling in Quantum Dots

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(Received 26 December 2002; published 9 September 2003)

Manifestations of quantum coherence in the electronic conductance through nearly closed quantum dots in the Coulomb-blockade regime are addressed. We show that quantum coherent tunneling processes explain some puzzling statistical features of the conductance peak heights observed in recent experiments at low temperatures. We employ the constant interaction model and the random matrix theory to model the quantum dot electronic interactions and its single-particle statistical fluctuations, taking full account of the finite decay width of the quantum dot levels.

DOI: 10.1103/PhysRevLett.91.116801

PACS numbers: 73.21.La, 03.65.Yz, 73.23.Hk

Recent experimental studies of electronic transport through nearly isolated quantum dots [1,2] assess the importance of quantum coherence and the nature of dephasing mechanisms in finite interacting electronic systems. Of particular interest is the Coulomb-blockade regime, where the thermal energy  $k_B T$  is much smaller than the charging energy  $E_C$  necessary to add an electron to the quantum dot. In this regime the conductance depends primarily on the quantum properties of the dot, such as its resonance levels and the corresponding line-widths due to the coupling between the dot and leads. Electrons are allowed to tunnel through the quantum dot whenever the charging energy is compensated by an external potential and the dot energy levels are in resonance with the chemical potential at the leads (small bias limit). The tunneling condition can be attained, for instance, by a tunable gate voltage  $V_g$ . In a typical experiment  $V_g$  is varied to obtain the conductance spectrum, a sequence of sharp (Coulomb-blockade) peaks.

Sequential tunneling is the key hypothesis for the standard rate equations [3] used to explain the transmission spectrum of quantum dots in the Coulomb-blockade regime [4,5]. This probabilistic picture neglects nonresonant quantum virtual processes, under the assumption that the resonant decay widths  $\Gamma$  are much smaller than both  $k_B T$  and the energy separation between the quantum dot resonances  $\delta\epsilon$ , namely,  $\Gamma \ll k_B T$  and  $\Gamma \ll \delta\epsilon$ , a condition often met by experiments in nearly isolated quantum dots.

The early experimental data taken from ballistic chaotic quantum dots were successfully confronted with the sequential theory by using the random matrix theory (RMT) to model the dot statistical single-particle properties [4,5]. More recently, the analysis of the measured conductance peak heights in the Coulomb-blockade regime [1,2] show significant deviations from this theory [6–8], indicating that some physics is missing. The inclusion of inelastic scattering processes [9–12], spin-orbit coupling [13], and exchange interaction [14,15] into the sequential approach expand in interesting ways the considered physical processes, adding new parameters to the

description. Unfortunately, these studies achieved only a limited success in reconciling theory with experiment.

In this Letter we show that quantum coherence, so far overlooked, leads to important corrections to the sequential tunneling picture [16] and explains some of the puzzles pointed out by the conductance experiments [1,2]. The importance of coherent processes is justified by noticing that while the sequential theory requires  $\Gamma \ll k_B T$ ,  $\delta\epsilon$ , the experiments satisfy those conditions only on average, namely,  $\langle \Gamma \rangle < \Delta \equiv \langle \delta\epsilon \rangle$  and  $\langle \Gamma \rangle \lesssim k_B T$ . Since both the decay width  $\Gamma$  and the resonance spacings  $\delta\epsilon$  fluctuate, conductance peaks where  $\Gamma$  is larger than  $k_B T$  and comparable to  $\delta\epsilon$  are not exceptional. More importantly, the study of fully coherent transport, as opposed to the sequential tunneling limit, provides a better framework to understand the interplay between coherence and interactions.

We describe a quantum dot coupled to external leads by the Hamiltonian

$$\hat{H} = \hat{H}_{\text{dot}} + \hat{H}_{\text{leads}} + \hat{H}_{\text{coupling}}. \quad (1)$$

We write the chaotic quantum dot Hamiltonian  $\hat{H}_{\text{dot}}$  as

$$\hat{H}_{\text{dot}} = \sum_j (E_j - e\eta V_g) d_j^\dagger d_j + \frac{e^2}{2C} \hat{N}(\hat{N} - 1), \quad (2)$$

where  $d_j^\dagger$  creates an electron in the  $j$ th eigenstate with energy  $E_j$  of the closed dot,  $\hat{N} = \sum_j d_j^\dagger d_j$  is the electron number operator in the dot,  $\eta V_g$  is the electrostatic energy due to the external gate (as usual,  $V_g$  is the gate voltage and  $\eta$  depends on the system specifics), and  $C$  is the effective dot capacitance. Equation (2) is the constant interaction model. In chaotic quantum dots ground state energy fluctuations due to interaction effects are very small in the large  $N$  limit [5]. We also do not account for spin and exchange interactions, which were recently addressed in the master equation framework by Refs. [14,15]. The electrons in the leads are treated as noninteracting, namely,

$$\hat{H}_{\text{leads}} = \sum_{k,a \in L,R} \epsilon_{k,a} c_{k,a}^\dagger c_{k,a} \quad (3)$$

where  $c_{k,a}^+$  creates an electron at the state of wave vector  $k = (2m^* \varepsilon_k)^{1/2}/\hbar$  at channel “a” either in the left ( $L$ ) or in the right ( $R$ ) lead. The dot-lead coupling term is

$$\hat{H}_{\text{coupling}} = \sum_{k,a \in L,R} \sum_j (V_{(k,a),j} c_{k,a}^+ d_j + \text{H.c.}) \quad (4)$$

The magnitude of the coupling matrix elements  $V_{(k,a),j}$  determines through a Fermi golden rule [17] the electron decay width  $\Gamma$  or the tunneling rate  $\Gamma/\hbar$  in the master equation framework. For quantum dots in the Coulomb-blockade regime ( $\Gamma$ ) is much smaller than the dot mean level spacing  $\Delta$ .

The conductance through the quantum dot is expressed in terms of the interacting system retarded Green’s function,  $G_{i,j}^R(t) = -(i/\hbar)\Theta(t)\langle\{d_i(t), d_j^+(0)\}\rangle$ . The evaluation of  $\langle G_{i,j}^R(t) \rangle$  follows the treatment presented by Baltin and collaborators [18] and generalizes their result to cases where the condition  $\Gamma \ll \delta\varepsilon$  is not met.

The retarded Green’s function is written as a sum over terms containing a different (and fixed) number of electrons in the dot

$$\langle G_{i,j}^R(t) \rangle = -\frac{i}{\hbar}\Theta(t) \sum_{N=0}^{\infty} P_N \langle \{d_i(t), d_j^+(0)\} \rangle_N, \quad (5)$$

where  $P_N$  is the thermal probability to find  $N$  electrons in the dot. This probability considers the full set of occupation numbers  $\{n_i\}$  of the  $\hat{H}_{\text{dot}}$  eigenstates. Equation (5) can be formally solved by the method of the equation of motion. In practice, the equations do not close unless we assume that the number of electrons in the dot does not fluctuate, which means that we replace  $\hat{N}$  by its expectation value  $N$  [19]. This simplification is entirely justified in the cases of interest, where  $e^2/C \gg \max(\Gamma, k_B T)$ .

The matrix representation of the retarded Green’s function is then cast as

$$\langle G^R(\varepsilon) \rangle = \sum_{N=0}^{\infty} P_N \{ [\varepsilon I - H_{\text{dot}}^{(N)} - \Sigma^R(\varepsilon)]^{-1} (I - n_N) + [\varepsilon I - H_{\text{dot}}^{(N-1)} - \Sigma^R(\varepsilon)]^{-1} n_N \}. \quad (6)$$

where the quantum dot matrix elements are

$$[H_{\text{dot}}^{(N)}]_{i,j} = (E_j - e\eta V_g + UN)\delta_{i,j}, \quad (7)$$

and  $U$  is the quantum dot charging energy, namely,  $U = e^2/C$ . In Eq. (6) we define  $[n_N]_{i,j} = \langle n_i \rangle_N \delta_{i,j}$  as the diagonal matrix whose entries are the canonical occupation numbers of the (closed) dot eigenstates. The retarded self-energy matrix elements, due to the coupling to the leads, become

$$[\Sigma^R(\varepsilon)]_{i,j} = \sum_{k,a \in L,R} \frac{V_{i,(k,a)} V_{(k,a),j}}{\varepsilon + i0^+ - \varepsilon_{k,a}}. \quad (8)$$

The coupling matrix elements  $V_{(k,a),j}$  vary in the energy scale of  $\varepsilon_k$  and hence are practically constant in energy windows comprising several single-particle states. We

neglect such variations to write

$$\Sigma^R(\varepsilon) = -\frac{i}{2}(\Gamma_L + \Gamma_R), \quad (9)$$

where  $\sum_k V_{i,(k,a)} V_{(k,a),j}/(\varepsilon + i0^+ - \varepsilon_{k,a}) = -i[\Gamma_a]_{i,j}/2$ . The energy dependence due to the principal value integral is negligible in the Coulomb-blockade regime.

The linear-response conductance is [17]

$$G = \frac{e^2}{h} g \quad \text{with } g = \int d\varepsilon \left( -\frac{\partial f_\mu}{\partial \varepsilon} \right) T_{R,L}(\varepsilon), \quad (10)$$

where  $f_\mu$  is the Fermi distribution function in the leads with chemical potential  $\mu$ .  $T_{R,L}$  is the system transmittance that can be computed from the retarded Green’s function

$$T_{R,L}(\varepsilon) = \left| \sum_{i,j} V_{(k,L),i} [G^R]_{i,j} V_{j,(k,R)} \right|^2. \quad (11)$$

Equivalently, the above expression can also be casted in the well-known form  $T_{R,L} = \text{tr}(\Gamma_R G^R \Gamma_L G^A)$  [17].

To this point our approach is quite general. Now, we replace  $G^R$  by its thermal average  $\langle G^R \rangle$ . Albeit restrictive [20], this is a very reasonable approximation for Coulomb-blockade peaks in quantum dots at low temperatures. Our approach is reduced to the sequential tunneling one [3] in the limit of  $\Gamma \ll \min(k_B T, \delta\varepsilon)$ . The main improvement is that we naturally account for quantum virtual tunneling processes. Those are significant whenever  $k_B T$  becomes comparable with  $\Gamma$ , a condition often met by experiments. Furthermore, both the single-particle level spacings  $\delta\varepsilon$  and the decay widths  $\Gamma$  fluctuate. Even if in average  $\Delta \gg \langle \Gamma \rangle$ , situations where  $\delta\varepsilon$  is comparable to  $\Gamma$  are not infrequent. In these cases quantum corrections are important. When the condition  $\Gamma/\delta\varepsilon \ll 1$  is always satisfied and not only in average, corrections to the conductance become indeed negligible. This was the limit analyzed in Ref. [18] for the phase lapse problem. Note also the contrast with the case of elastic cotunneling at the conductance valleys. There, the contribution of the off-resonant levels is of order  $\Gamma/U$ , whereas here their contribution is of order  $\Gamma/\delta\varepsilon$ .

We switch now to the statistical study of the dimensionless conductance peak heights  $g^{\text{max}}$ . This analysis allows for a comparison between the results of our approach, experiments, and the sequential tunneling theory. The statistical ansatz is to assume that the underlying electronic dynamics in the quantum dot is very complex, and hence the fluctuation properties of its single-particle eigenenergies and eigenfunctions coincide with those of an ensemble of random matrices [4,5]. Accordingly, the single-particle levels display universal fluctuations and their spacings  $\delta\varepsilon$  follow the Wigner-Dyson distribution. Likewise, the decay widths  $\Gamma$  are Porter-Thomas distributed. The physical inputs of the statistical theory are only the mean level spacing  $\Delta$  and the average decay width  $\langle \Gamma \rangle$ . We consider the dot both in the absence of a magnetic field (orthogonal ensemble,  $\beta = 1$ ) and in the presence of

a magnetic field that breaks the time-reversal symmetry (unitary ensemble,  $\beta = 2$ ). The later is the relevant one to be compared with the available experimental data.

The numerical implementation is straightforward but costly since Eq. (6) requires matrix inversions for each realization. The canonical thermal quantities  $P_N$  and  $\langle n_i \rangle_N$  are computed using the quadrature formula explained in Ref. [21], which was already used for quantum dots [7,8]. For  $k_B T \lesssim \Delta$  good accuracy requires taking into account at least 30 levels around the resonant one. We use typically  $10^5$  realizations for the ensemble averaging. The charging energy  $U$  is taken to be  $50\Delta$  (the results are quite insensitive to  $U$ , provided  $U \gg \Delta$ ).

The data of Ref. [1] show that at very low temperatures,  $k_B T \ll \Delta$ , the conductance peak height distribution does not follow the standard random matrix theory [6]. By accounting for quantum coherent tunneling we obtain a very nice agreement with the experimental distributions. This is illustrated in Fig. 1 for  $B \neq 0$  ( $\beta = 2$ ). In the inset we present our results for the distribution of  $g^{\max}$  for  $B = 0$  ( $\beta = 1$ ). In Fig. 1 the dimensionless conductance peak heights  $g^{\max}$  are scaled to the unit mean. We show the peak heights distribution for  $k_B T = 0.1\Delta$ ,  $\langle \Gamma \rangle = 0.1\Delta$  (solid line), and  $\langle \Gamma \rangle = 0.2\Delta$  (dashed line). The histogram corresponds to the experimental result of Ref. [1] available only for  $B \neq 0$  ( $\beta = 2$ ). Different dots have different  $\langle \Gamma \rangle / \Delta$ , a ratio that can be determined from the experimental  $g^{\max}$ .  $\langle \Gamma \rangle / \Delta \sim 0.1$  is representative of the analyzed experiments. We find that as the ratio  $\langle \Gamma \rangle / \Delta$  is increased, the probability to obtain small conductances is suppressed in comparison with the standard sequential theory (dotted line). This can be understood as follows: If a given resonance has small tunneling rates, the contributions due to virtual processes through its neighbors will reduce the chance to obtain a very small peak. Thus, we expect  $P(g^{\max} = 0) = 0$ .

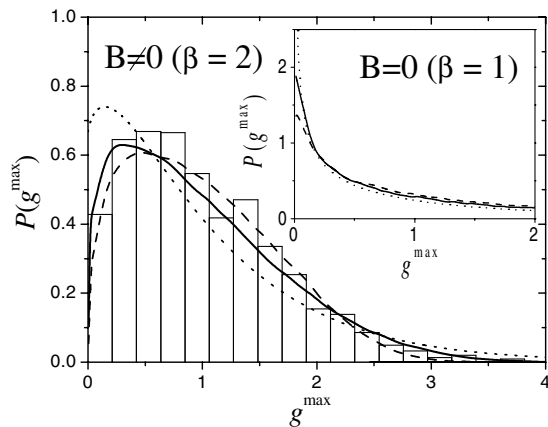


FIG. 1. Peak height probability distribution  $P(g^{\max})$  for  $k_B T = 0.1\Delta$  and  $B \neq 0$  ( $\beta = 2$ ). The same for  $B = 0$  ( $\beta = 1$ ) in the inset. Our theory for  $\langle \Gamma \rangle / \Delta = 0.1$  (solid line) and  $0.2$  (dashed line) is compared with the standard sequential tunneling result (dotted line), and the experimental distribution (histogram) [1].

In the early experiment by Chang *et al.* [22] special care was taken to discard from the statistical sample conductance peak heights that did not fulfill  $\Gamma \ll k_B T$ . Hence, corrections due to the finite ratio  $\Gamma / \Delta$  are practically negligible. This might explain why a good agreement with the standard sequential theory was found there [22]. Note also that as  $k_B T$  becomes comparable with  $\langle \Gamma \rangle$  the assessment of the quantum dot temperature through the widths of the Coulomb-blockade peaks becomes unreliable, due to the non-negligible  $\Gamma$ .

The experimental results of Ref. [1] show another striking and unexplained discrepancy with respect to the standard rate equations. This is best quantified by the ratio between the standard deviation  $\delta g^{\max}$  and the mean conductance peak heights  $\langle g^{\max} \rangle$ , namely,

$$\sigma_g = \frac{\delta g^{\max}}{\langle g^{\max} \rangle} = \frac{\sqrt{\langle (g^{\max})^2 \rangle - \langle g^{\max} \rangle^2}}{\langle g^{\max} \rangle}. \quad (12)$$

In the experiments  $\delta g^{\max}$  is significantly smaller than predicted by the rate equations plus RMT. Recent works [9,11,12] discuss whether such deviations can be attributed to inelastic processes [23]. Our approach explains the experimental findings in the low temperature regime  $k_B T / \Delta \ll 1$ , where inelastic processes are hard to justify. In Fig. 2 we show  $\sigma_g$  for  $B \neq 0$  ( $\beta = 2$ ) as a function of the thermal energy for different values of  $\langle \Gamma \rangle / \Delta$ . The inset shows  $\sigma_g$  for the case when  $B = 0$  ( $\beta = 1$ ). The standard sequential theory results [7] are illustrated by the dotted lines.

At low temperatures and as  $\langle \Gamma \rangle / \Delta$  is increased, our  $\sigma_g$  is significantly reduced with respect to the standard sequential theory prediction. For higher temperatures,  $k_B T \gtrsim 0.5\Delta$ , we obtain larger  $\sigma_g$  than the measured ones. Furthermore, as the temperature increases, our  $\sigma_g$

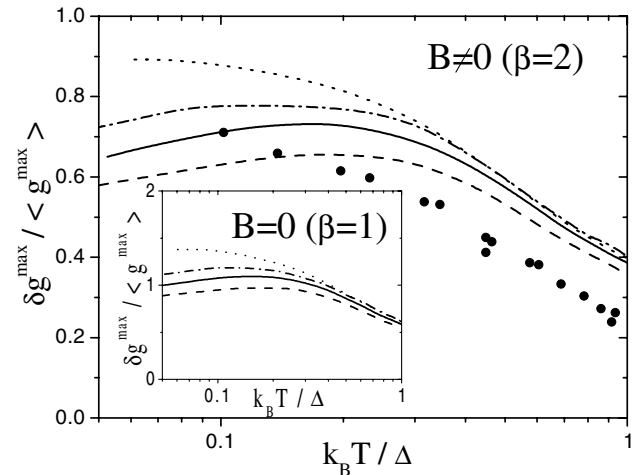


FIG. 2. Normalized peak heights distribution width  $\sigma_g$  for  $B \neq 0$  (the  $B = 0$  case is shown in the inset) as a function of  $k_B T / \Delta$ , for  $\langle \Gamma \rangle / \Delta = 0.05, 0.1, 0.2$  (dash-dotted, solid, and dashed lines, respectively). Symbols correspond to the experimental results of Ref. [1] for different dots and the dotted lines to the standard sequential theory.

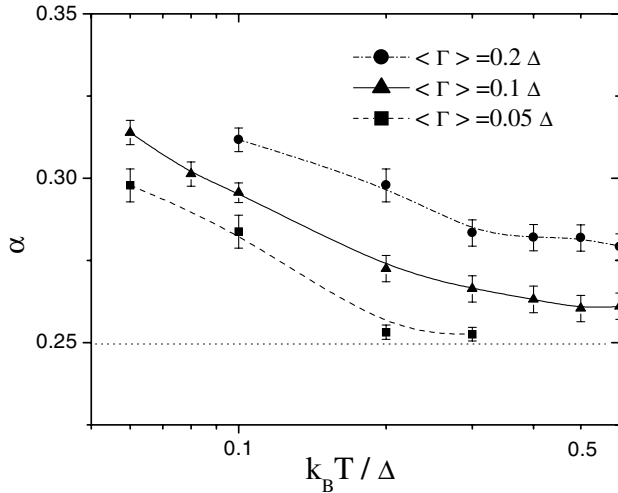


FIG. 3. Normalized change in the average conductance  $\alpha$  as a function of temperature for different  $\langle \Gamma \rangle / \Delta$ .

approaches the standard theory result. Similar behavior was also recently found by including the exchange term in  $\hat{H}_{\text{dot}}$  [14,15]. However, at high temperatures we expect a reduction of the peak heights fluctuations due to inelasticity and decoherence.

The suppression of the weak localization peak was recently used to determine the dephasing time  $\tau_\phi$  in open quantum dots [24,25]. This inspired Folk *et al.* to experimentally investigate the change in the conductance peak height upon breaking the time-reversal symmetry of the quantum dots by applying a magnetic field  $B$ , namely,

$$\alpha = \frac{\langle g^{\text{max}} \rangle_{B \neq 0} - \langle g^{\text{max}} \rangle_{B=0}}{\langle g^{\text{max}} \rangle_{B \neq 0}}. \quad (13)$$

At zero temperature the sequential tunneling theory gives a constant  $\alpha = 1/4$ . Inclusion of temperature corrections and spectral fluctuations gives small changes, essentially keeping  $\alpha \approx 1/4$  [11,12]. In Fig. 3 we show  $\alpha$  as a function of temperature for different values of  $\langle \Gamma \rangle / \Delta$ . Our simulations show that  $\alpha$  is larger than  $1/4$  at low temperatures and decreases with increasing  $k_B T$ . This behavior suggests that a finite ratio  $\langle \Gamma \rangle / \Delta$  enhances more effectively the conductance in the unitary case than in the orthogonal case. Since  $\alpha$  is very sensitive to the ratio  $\langle \Gamma \rangle / \Delta$ , particular care must be exercised when comparing data corresponding to different quantum dots. As in the analysis of  $\sigma_g$  our results suggest that an additional physical process is needed to explain the experimental data for  $k_B T \gtrsim \Delta$ .

In summary, we have investigated the effect of quantum coherent processes on the statistics of the conductance peak heights. We found that at very low temperatures this leads to significant corrections to the distribution of conductance peak heights obtained using the standard sequential theory. The relevant parameter for

these corrections is  $\langle \Gamma \rangle / k_B T$ . Our study also indicates that estimates of the inelastic scattering rates and the strength of the effective exchange interaction in quantum dots using the peak height distributions need to account for coherent tunneling in order to be quantitative.

L. E. F. T. and C. H. L. thank the CBPF for the hospitality. This work was supported by Antorchas, Vitae, CONICET, SeCyT-UNC, Instituto do Milênio de Nanociências, and CNPq.

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